

Calculus II
Practice Problems 13

1. A man drops out of a plane at 25,000 feet of altitude and immediately opens his parachute. For this man and parachute the deceleration due to air resistance is $4v$ where v is his velocity. How far has he fallen in one minute? How long does it take for him to hit the ground, and at approximately what velocity does he hit the ground?
2. a) Let a mass m hang from a spring of spring constant k . Suppose that it is set in motion. Show that, throughout the motion, $mv^2 + kx^2$ is constant, where x represents displacement from equilibrium, and v is velocity.
b) Suppose that $k = 4$ dynes/cm and $m = 10$ g, and the spring is already in motion. At a particular instant the spring is located 10 cm. from equilibrium and traveling at velocity 8 cm/sec. For this motion, what are the maximum velocity and maximum displacement of the mass?
3. The above configuration is put in a viscous fluid which exerts a retardant force proportional to the velocity, with constant of proportionality $q = 12$. Find the equation of motion of the mass, given that at time $t = 0$ it is at $x = 0$ and its velocity is 4.8 cm/sec. What is the maximum displacement of the mass?
4. A crystal glass consists of cells in a crystalline shape which oscillate at a natural frequency, so the motion is governed by a differential equation $x'' + \omega_0^2 x = 0$ where $2\pi/\omega_0$ is the frequency. If the ambient air is vibrating at a frequency of $2\pi/\omega$ (due to a monotonal sound, perhaps), then the motion of a crystal is modified by the force of the air in motion so as to be governed by the inhomogeneous equation

$$x'' + \omega_0^2 x = A \cos \omega t$$

Find a particular solution of this differential equation. What happens as ω approaches ω_0 ? (This phenomenon is called *resonance*.)

5. Consider the circuit as shown in the diagram. The switch is turned on at time $t = 0$. Find the current as a function of time.