Calculus II
Problems on Numerical Methods

1. Since $\tan(\pi/6) = 1/\sqrt{3}$,and therefore, $\pi = 6\arctan(1/\sqrt{3})$ we can use the Taylor series for the arc tangent to estimate π. Do this, using the first three nonzero terms.

2. Since $\sin(\pi/6) = 1/2$, we can also find π by solving the equation $\sin x = 1/2$. We can approximate the solution by replacing sin by an approximating Taylor polynomial, and then using Newton's method. Do this with the three term Taylor polynomial for $\sin x$.

3. Find a solution, by Newton's method, of the equation

$$x^5 - x^4 + x^3 - x^2 = 4$$

correct to five decimal places.

4. Here is another way of estimating π. We know that

$$\pi/4 = \int_0^1 \frac{dx}{1+x^2} \ .$$

Estimate this integral by the trapezoid rule, using steps of size $1/10$. How many steps should we take to be sure of an estimate correct to 4 decimal places?

5. Define

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4^n (n!)(n+1)!} \ .$$

Evaluate $J_0(1)$ correctly to 4 decimal places.

6. Find an estimate for

$$\int_0^2 \frac{\sin x}{x} dx$$

using Simpson' s rule with $N = 20$ subdivisions.