MATH 1220-90 Fall 2011

Third Midterm Exam

INSTRUCTOR: H.-PING HUANG

LAST NAME ________________________

FIRST NAME Grader's Copy

ID NO. ________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	20	
PROBLEM 2	20	
PROBLEM 3	20	
PROBLEM 4	20	
PROBLEM 5	20	
TOTAL	100	

PROBLEM 1

(20 pt) Consider the sequence

$$a_n = \frac{\ln(1/n)}{\sqrt{2n}}.$$

What is $\lim_{n\to\infty} a_n$?

~~(5 pt)~~

$$\lim_{n\to\infty} \frac{\ln\left(\frac{1}{n}\right)}{\sqrt{2n}} \quad \frac{-\infty}{\infty}$$

L'Hopital

(5 pt)

$$\ln\left(\frac{1}{n}\right) = \ln\left(n^{-1}\right) = -\ln(n)$$

$$\sqrt{2n} = (2n)^{+\frac{1}{2}} = \sqrt{2}\, n^{+\frac{1}{2}}$$

$$= \lim_{n\to\infty} \frac{-\frac{1}{n}}{\sqrt{2}\left(\frac{1}{2}\right) n^{-\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{-1}{\sqrt{2}\left(\frac{1}{2}\right)} \cdot \frac{1}{n^{0.5}}$$

(~~0~~ pt) 10

$= 0$ (5 pt)

use quotient so that get the same resut → ~~(15 pt)~~

same pts

PROBLEM 2

(20 pt) Use the Integral Test to decide the convergence or divergence of the following series:

$$\sum_{n=1} \frac{n}{e^n}.$$

$$\int_{x=1}^{\infty} x e^{-x} \, dx \qquad (5pt)$$

integration by parts

$$= \underbrace{-x e^{-x} \Big|_1^{\infty} - \int_1^{\infty} (-e^{-x})\, dx}_{(5pt)}$$

(improper intogal)

$$= -x e^{-x} - e^{-x} \Big|_1^{\infty} \qquad (5pt)$$

$$= \lim_{n \to \infty} -\frac{x}{e^x} - e^{-x} + [e^{-1} + e^{-1}]$$

$$= 0 + \frac{2}{e} = \frac{2}{e} \quad \text{Conv.}$$

L'Hopital
[Rule in fact]

(5pt)

PROBLEM 3

(20 pt) Decide the convergence or divergence of the following series:

$$\sum_{n=1}^{\infty} \frac{2(6)^n}{9^{2n}}$$

If it is convergent, find its sum. If not, prove it.

Geometric Series (5 pt)

$$\text{ratio} = \frac{6}{9^2} = \frac{2}{27} < 1$$

(5 pt)

conv. (5 pt)

$$\text{Sum} = \frac{\frac{2 \cdot 6}{9^2}}{1 - \frac{2}{27}}$$

(5 pt)

PROBLEM 5

(20 pt) Find the power series representation for $f(x) = xe^{x^2}$. What is the set of convergence?

power series $= x \left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots\right)$

(5pt)

center of conv. $\Rightarrow$ $x = 0$

$$\left[\sum_{n=1}^{\infty} \frac{x^{2n+1}}{n!}\right]$$

(5pt)

inverse of radius : $\lim_{n\to\infty} \left| \dfrac{\frac{x^{2n+3}}{(n+1)!}}{\frac{x^{2n+1}}{n!}} \right| = \left| \frac{x^2}{n+1} \right|$

$= \lim_{n\to\infty} |x^2| \frac{1}{n+1}$

$= 0$

(10pt) radius $= \infty$

set of conv. = all real numbers

PROBLEM 4

(20 pt) Find the power series representation for

$$f(x) = \frac{1}{(1+x)^2}$$

and specify the radius of convergence.

Two methods.

(1) ~~binomial~~

not easy

(2) $f(x) = \frac{d}{dx}\left(\frac{-1}{1+x}\right)$ (5pt) $\quad a_1 = -1$, ratio $= -x$ (5pt)

$$= \frac{d}{dx}\left[-1 + x - x^2 + x^3 - x^4 + \cdots\right]$$

$$f(x) = 1 - 2x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots$$

[important $f(0)$] (5pt)

radius of conv. inherited from

$-\frac{1}{1+x}$ (5pt) $\quad |-x| < 1$

$|x| < 1$

radius $= 1$