MATH 1220-90 Fall 2011
Third Midterm Exam
INSTRUCTOR: H.-PING HUANG

LAST NAME ________________________________
FIRST NAME _______ Grader's Copy _______
ID NO. ________________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE SPECIFIED METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1  20 _______
PROBLEM 2  20 _______
PROBLEM 3  20 _______
PROBLEM 4  20 _______
PROBLEM 5  20 _______
TOTAL 100 _______
PROBLEM 1

(20 pt) Consider the sequence

$$a_n = \frac{\ln(1/n)}{\sqrt{2n}}.$$ 

What is $\lim_{n \to \infty} a_n$?

\[
\lim_{n \to \infty} \frac{\ln \left( \frac{1}{n} \right)}{\sqrt{2n}} \quad \overset{\text{L'Hopital's}}{=} \quad \lim_{n \to \infty} \frac{- \frac{1}{n}}{\frac{1}{\sqrt{2n}}} = \lim_{n \to \infty} \frac{- \frac{1}{n}}{\frac{1}{2 \sqrt{n}}} = \lim_{n \to \infty} -\frac{1}{n^{0.5}} = 0
\]

Use quotient so that get the same result $\Rightarrow$ 

same pts
PROBLEM 2

(20 pt) Use the Integral Test to decide the convergence or divergence of the following series:

\[ \sum_{n=1}^{\infty} \frac{n}{e^n} \]

\[ \int_{\infty}^{1} x e^{-x} \, dx \quad (5 \text{ pt}) \]

Integration by parts

\[ = \left[ -xe^{-x} \right]_{1}^{\infty} - \int_{1}^{\infty} (-e^{-x}) \, dx \]

(Improper integral)

\[ = \lim_{n \to \infty} \left[ -xe^{-x} \right]_{e}^{n} + \left[ e^{-x} + e^{-1} \right] \]

\[ = 0 + \frac{2}{e} = \frac{2}{e} \quad \text{Conv} \]

L'Hôpital's Rule in fact
PROBLEM 3

(20 pt) Decide the convergence or divergence of the following series:

\[ \sum_{n=1}^{\infty} \frac{2(6)^n}{9^{2n}} \]

If it is convergent, find its sum. If not, prove it.

Geometric Series (5 pt)

\[ \text{ratio} = \frac{6}{9^2} = \frac{6}{27} = \frac{2}{9} < 1 \]

Conv. (5 pt)

\[ \text{Sum} = \frac{2 \cdot 6}{9^2} \]

\[ \frac{1 - \frac{2}{27}}{1} \] (5 pt)
PROBLEM 5

(20 pt) Find the power series representation for \( f(x) = xe^{x^2} \). What is the set of convergence?

\[
\text{Power series} = x \left( 1 + x + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots \right)
\]

\[
\text{Center of conv.} \Rightarrow x = 0 \quad (5 \text{ pt})
\]

\[
\sum_{n=1}^{\infty} \frac{x^{2n+3}}{n!} = \left( \sum_{n=0}^{\infty} \frac{x^{2n+3}}{(n+1)!} \right) \quad \text{(5 pt)}
\]

\[
\text{Inverse of radius} \quad \lim_{n \to \infty} \left| \frac{x^{2n+3}}{(n+1)!} \right| = \frac{x^2}{n+1}
\]

\[
\text{Radius} = \infty \quad (10 \text{ pt})
\]

\[
\text{Set of conv.} = \text{all real numbers}
\]
PROBLEM 4

(20 pt) Find the power series representation for

\[ f(x) = \frac{1}{(1 + x)^2} \]

and specify the radius of convergence.

Two methods.

1. Not easy

\[ f(x) = \frac{1}{(1 + x)^2} \]

\[ a_1 = -1 \]

\[ \text{ratio} = -x \]

\[ = \frac{d}{dx} \left[ -1 + x - x^2 + x^3 - x^4 + \ldots \right] \]

\[ = \frac{d}{dx} \left[ f(x) \right] \]

\[ \text{important} \]

\[ f(0) \]

radius of conv. inherited from

\[ \frac{-1}{1 + x} \]

\[ (-x) \]

\[ 1 \times 1 < 1 \]

radius = 1