R MATH 1220-9 Exam # 2

Problem 1

$\int x \ln(2x)\, dx$ (use intergration by parts)

$\int u\,dv = uv - \int v\,du$

Let $u = \ln(2x)$ $dv = x\,dx$ (5pts)

$du = \frac{1}{2x} \cdot 2\,dx = \frac{1}{x}dx$ $v = \frac{1}{2}x^2$ (5pts)

So $\int x \ln(2x)\, dx = \ln(2x) \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$ (5pts)

$= \frac{1}{2}x^2 \ln(2x) - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2 \ln(2x) - \frac{1}{4}x^2 \left(+C\right)$ (5pts

Problem 2

$$\int_1^\infty \frac{1}{x^p}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^p}\,dx \overset{(p\neq 1)}{=} \lim_{b\to\infty}\left(\frac{1}{1-p}x^{1-p}\Big|_1^b\right)$$

(5pts)

$$= \lim_{b\to\infty}\frac{1}{1-p}\left(b^{1-p}-1\right)$$

Hence, $p<1 \Rightarrow 1-p>0 \Rightarrow \lim_{b\to\infty} b^{1-p} = \infty$

intergral $\int_1^\infty \frac{1}{x^p}dx$ diverges (5pts)

$p>1 \Rightarrow 1-p<0 \Rightarrow \lim_{b\to\infty} b^{1-p} = 0$

intergral $\int_1^\infty \frac{1}{x^p}dx$ converges (5pts)

$p=1$ $\int_1^\infty \frac{1}{x}dx = \lim_{b\to\infty}\int_1^b \frac{1}{x}dx = \lim_{b\to\infty} \ln|x|\Big|_1^b = \infty$

diverges (5pts)

Problem 3

$$\int \sin^5 x \, dx$$

$$= \int \sin^4 x \cdot \underline{\sin x \, dx}$$ (5pts)

$$= \int (1-\cos^2 x)^2 \, \underline{d(-\cos x)}$$ (5pts)

$$= -\int (1-\cos^2 x)^2 \, d(\cos x)$$

Let $u = \cos x$ $\quad du = d(\cos x)$

$$= -\int (1-u^2)^2 \, du = -\int (u^4 - 2u^2 + 1) \, du$$ (5pts)

$$= -\frac{1}{5}u^5 + \frac{2}{3}u^3 - u \; (+\, C)$$

$$= -\frac{1}{5}\cos^5 x + \frac{2}{3}\cos^3 x - \cos x \; (+\, C)$$ (5pts)

Problem 4

$$\frac{1}{x^2+x-6} = \frac{1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \quad \text{(5pts)}$$

So $\quad 1 = A(x+3) + B(x-2) \quad$ (5pts)

$(x=2) \quad A = \frac{1}{5}$

(5pts)

$(x=-3) \quad B = -\frac{1}{5}$

$$\int \frac{1}{x^2+x-6}\,dx = \int \left( \frac{A}{x-2} + \frac{B}{x+3} \right) dx$$

$$= \int \frac{A}{x-2}\,dx + \int \frac{B}{x+3}\,dx$$

(5pts) $\quad = A \ln|x-2| + B \ln|x+3| \ (+C)$

or $= \frac{1}{5}\ln|x-2| - \frac{1}{5}\ln|x+3| \ (+C)$

Problem 5

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

$$\overset{\left(\frac{0}{0}\right)}{\underset{(5pts)}{=}} \lim_{x \to 0} \frac{\cos x - 1}{3x^2} \quad \text{(5pts)}$$

$$\overset{\left(\frac{0}{0}\right)}{=} \lim_{x \to 0} \frac{-\sin x}{6x} \quad \text{(5pts)}$$

$$\overset{\left(\frac{0}{0}\right)}{=} \lim_{x \to 0} \frac{-\cos x}{6} = \boxed{-\frac{1}{6}} \quad \text{(5pts)}$$