MATH 1220-90 Fall 2011

First Midterm Exam

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LAST NAME _____

FIRST NAME Grader's Copy

ID NO. _____

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	20	_____
PROBLEM 2	20	_____
PROBLEM 3	20	_____
PROBLEM 4	20	_____
PROBLEM 5	20	_____
TOTAL	100	_____

PROBLEM 1

(20 pt) Find

$$\int_1^8 \frac{\ln(3x)}{x}\,dx.$$

(5 pt)

$u = \ln(3x)$ $\quad du = \frac{1}{3x}\,3\,dx$

$= \frac{1}{x}\,dx$

($\ln(3x) = \ln 3 + \ln x$)

5 pt

$$\int_{\ln 3}^{\ln 24} u\,du$$

(5 pt) ← $u = \ln 3$

$$= \frac{1}{2}u^2 \Big|_{\ln 3}^{\ln 24}$$

(5 pt)

PROBLEM 2

(20 pt) A curve is given by the equation:

$$y^3 + 129 = (e^x + 1)^2.$$

Find the slope of the tangent line at the point $(0, -5)$.

$3y^2 \frac{dy}{dx} = 2(e^x + 1)e^x$ (10 pt)

(5 pt)

$3 \cdot (-5)^2 \, y' = 2(1+1) \cdot 1$

$y' = \frac{2 \cdot 2}{3 \cdot 25}$ (5 pt)

[In fact, it should be checked first that

$(-5)^3 + 129 = (e^0 + 1)^2$]

PROBLEM 3

(20 pt) Find

$$\int \frac{x^2 - x}{x+1} dx.$$

5 pt

$$I = \int x - 2 + \frac{2}{x+1} \, dx$$

$$= \frac{1}{2} x^2 - 2x + 2 \ln |x+1|$$

(5 pt)

(10 pt) including the absolute value sign

PROBLEM 4

(20 pt) $x > 0$, find $D_x y$ if

$$y = (6x)^{6x}.$$

(1) $\ln y = 6x \ln(6x)$ (5 pt)

$\frac{1}{y} \frac{dy}{dx} = 6 \ln(6x) + 6x \cdot \frac{1}{x}$ (5 pt) (5 pt)

$\frac{dy}{dx} = \left[6 \ln(6x) + 6\right](6x)^{6x}$ (5 pt)

or

(2) $y = e^{6x \ln(6x)}$ (5 pt)

$y' = e^{6x \ln(6x)}$ → (5 pt) (10 pt)

↓ ↗

equal to $(6x)^{6x}$ $\quad [6 \ln(6x) + 6]$

PROBLEM 5

(20 pt) An object is taken from an oven at 350°F and left to cool in a room at 70°F. If the temperature fell to 250°F in one hour, what would its temperature be three hours after it was removed from the over?

$t = T - 70$, ($5t$)

$\frac{dt}{dh} = kt$ ($5t$)

$t_1 = t_0 e^{k \cdot h}$

$(250 - 70) = (350 - 70) e^{k \cdot 1}$

$k = \ln \frac{180}{280}$ ($5pt$)

Jia, they can use this form.

$t_3 = t_0 e^{3 \ln \frac{180}{280}}$ ($5pt$)

$= (350 - 70)(\frac{180}{280})^3$

$T = t_3 + 70$ (°F) (1 pt off. °F) without