#1 Change $f^{-1}(7)$ to $(f^{-1})'(7)$

MATH 1220-90 Fall 2011

Final Exam

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Hint: do NOT calculate any numerical value, unless specified otherwise.

LAST NAME ______________________

FIRST NAME Grader's Copy

ID NO. ______________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	30	______
PROBLEM 2	30	______
PROBLEM 3	30	______
PROBLEM 4	30	______
PROBLEM 5	30	______
PROBLEM 6	20	______
PROBLEM 7	20	______
PROBLEM 8	30	______
TOTAL	160	______

change $f^{-1}(7)$ to $(f^{-1})'(7)$

PROBLEM 1

$(f^{-1})'(7)$

(30 pt) Let $f(x) = 2 + 3x + 5e^x$. Find $f^{-1}(7)$.

$f^{-1} : y \to x$

$y = 7$ $\qquad 2 + 3x + 5e^x = 7$

(10pt) $\qquad x = 0$

(10pt)

$$(f^{-1})' = \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

$$= \frac{1}{3 + 5e^x} \Big|_{x=0} = \frac{1}{8}$$

(10pt)

PROBLEM 2

(30 pt) Use integration by parts to evaluate the integral.

$$\int xe^{2x}dx$$

$$\frac{d\,x}{dx} = 1$$

$$\int e^{2x} = \frac{1}{2} e^{2x}$$ (10pt)

(10pt)

$$\int x e^{2x} dx = x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} \cdot e^{2x} dx$$

$$= x \cdot \frac{1}{2} \cdot e^{2x} - \frac{1}{4} e^{2x}$$ (10pt)

PROBLEM 4

(30 pt) Find the slope of the tangent to the curve $r = 9 + 2\cos\theta$ at the value $\theta = \pi/2$.

$\theta = \frac{\pi}{2}$ $\quad r = 9$ (5pt)

(5pt)

$dy/dx = \frac{2}{9}$

$y = r\sin\theta = 9\sin\theta + 2\cos\theta\sin\theta$

$[= 9\sin\theta + \sin(2\theta)]$

$\frac{dy}{d\theta} = 9\cos\theta + 2\cos(2\theta)\Big|_{\theta=\frac{\pi}{2}}$

(10pt)

$= -2$

$x = r\cos\theta$

$[= 9\cos\theta + 1 + \cos(2\theta)]$

(10pt)

$\frac{dx}{d\theta} = -9\sin\theta - 2\sin(2\theta)\Big|_{\theta=\frac{\pi}{2}}$

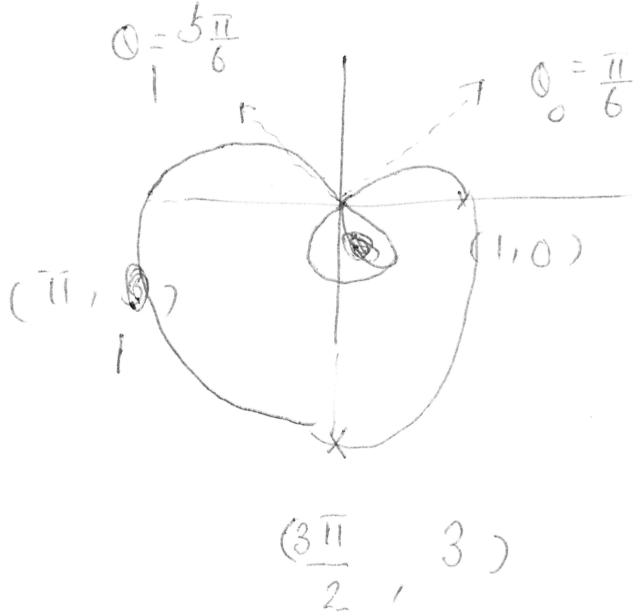
$= -9$

PROBLEM 5

(20 pt) Find the area inside the inner loop of the following limaçon: $r = 1 - 2\sin\theta$.

$r = 0 \quad \sin\theta = \frac{1}{2}$ (10pt)

$\theta_0 = \frac{\pi}{6}, \quad \frac{5\pi}{6} = \theta_1$



$[\sin\theta_0 = \frac{1}{2} \qquad \sin(2\theta_0) = \frac{\sqrt{3}}{2}$

$\cos\theta_0 = \frac{\sqrt{3}}{2}$

$\sin\theta_1 = \frac{1}{2} \qquad \sin(2\theta_1)$

$\cos\theta_1 = -\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}]$

(10pt) $A = \int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}(1 - 2\sin\theta)^2 \, d\theta$

(10pt) $= \int \frac{1}{2}(1 - 4\sin\theta + 2(1 - \cos 2\theta))\, d\theta$

$= \frac{1}{2}(\theta + 4\cos\theta + 2(\theta - \frac{1}{2}\sin 2\theta))\Big|_{\theta_0}^{\theta_1}$

PROBLEM 6

(20 pt) Solve the following differential equation:

$$y'' + 9y = 0; \quad y = 3, \text{ and } y' = 3 \text{ at } x = \frac{\pi}{3}.$$

$D^2 + 9 = 0$

$[D = \pm 3i]$

(10pt)

$y = a\cos(3x) + b\sin(3x)$

$y' = -3a\sin(3x) + 3b\cos(3x)$

$x = \frac{\pi}{3}$ $\quad 3x = \pi$ $\quad \cos(\pi) = -1$

$\sin(\pi) = 0$

$-a = 3$

$-3b = 3$ $\quad$ (10pt)

$b = -1$

PROBLEM 7

(30 pt) Solve the following differential equation:

$$y'' - 3y' - 10y = 0; \quad y = 1, \text{ and } y' = 10 \text{ at } x = 0.$$

$D^2 - 3D - 10 = 0$

$(D-5)(D+3) = 0$ (10pt)

$D = 5$ or -3

$y = ae^{5x} + be^{-3x}$

$y' = 5ae^{5x} - 3be^{-3x}$

$X = 0$ $\quad a + b = 1$ (10pt)

$5a - 3b = 10$

$a = \frac{13}{8}$ $\quad b = -\frac{5}{8}$

PROBLEM 8

(30 pt) Determine the distance between the vertices of

$$-9x^2 + 18x + 4y^2 + 24y - 9 = 0.$$

$$(-9x^2 + 18x) + (4y^2 + 24y) = 9$$

$$-9(x^2 - 2x) + 4(y^2 + 6y) = 9$$

(10pt) $$-9\left[(x-1)^2 - 1\right] + 4\left[(y+3)^2 - 3^2\right] = 9$$

$$-(x-1)^2 + 4(y+3)^2 = 4 \cdot 3^2$$

(5pt) $$-\frac{(x-1)^2}{6^2} + \frac{(y+3)^2}{3^2} = 1$$

(5pt)

distance $= 2a \; [= 6]$