Calculus II
Practice Exam 3

In problems 1-4, find the limits.

1. $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2}$

2. $\lim_{x\to \pi} \dfrac{(x-\pi)^3}{\sin x + x - \pi}$

3. $\lim_{x\to \infty} x^5 e^{-x}$

4. $\lim_{x\to \infty} \dfrac{\sqrt{1+x^2} - x}{x}$

In problems 5-7: Does the integral converge or diverge. If you can, find the value of the integral.

5. $\int_0^\infty x e^{-x^2} dx$

6. $\int_0^\infty \dfrac{x^2}{x^3+1} dx$

7. $\int_0^1 \dfrac{dx}{x^{9/10}}$

8. Does the sequence converge or diverge?

$$\text{a)} \quad a_n = \frac{n^2}{n!}, \quad \text{b)} \quad b_n = \frac{\sqrt{n!}}{(n+1)^2}, \quad \text{c)} \quad c_n = \frac{n^3 - 50n + 1}{n^4 + 123n^3 + 1}$$

9. Does the series converge or diverge?

$$\text{a)} \quad \sum_1^\infty \frac{n^2}{n!}, \quad \text{b)} \quad \sum_1^\infty \frac{\sqrt{n!}}{(n+1)^2}, \quad \text{c)} \quad \sum_{20}^\infty \frac{n^3 - 50n + 1}{n^4 + 123n^3 + 1}$$

10. Does the series converge or diverge?

$$\text{a)} \quad \sum_1^\infty \frac{3n+1}{n^{5/2}}, \text{b)} \quad \sum_1^\infty \frac{3^n n!}{(n+1)!5^n + 1}, \text{c)} \quad \sum_1^\infty \frac{(2n)!(n+1)}{(2n+1)!}, \text{d)} \quad \sum_1^\infty \frac{1}{n^{1/2}(3n+1)}$$

11. Find the radius of convergence of the series:

$$\text{a)} \quad \sum_{n=3}^\infty n(n-1)(n-2)x^{n-3}, \text{b)} \quad \sum_0^\infty (2^n + 1)x^n, \text{c)} \quad \sum_1^\infty \left(\frac{3n^2+1}{n^3+1}\right)(x+1)^n$$

12. Find the Maclaurin series for $(1+x)^{-3}$.

13. Find the Maclaurin series for $\int_0^x \arctan t\, dt$.

14. Find the Maclaurin series for $x\ln(x+1)$.

15. Find the terms up to fourth order for the Maclaurin series for

$$\frac{e^x}{1+x}$$