# Calculus II 1220-90 Exam 2

**Summer 2014**

Name ______________________________________________

**Instructions.** Show all work and include appropriate explanations when necessary. A correct answer without accompanying work may not receive full credit. Please try to do all all work in the space provided and circle your final answer.

1. (20pts) Evaluate the following limits using L'Hôpital's rule or any other method. Note, the correct answer may be $\pm\infty$.

   (a) (5pts) $\displaystyle\lim_{x\to\infty} \frac{2x^2+x+1}{5x^2-2x}$

   (b) (5pts) $\displaystyle\lim_{x\to 0^+} \frac{\cos x - \sin x}{x}$

   (c) (5pts) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{x-\frac{\pi}{2}}{\cos x}$

   (d) (5pts) $\displaystyle\lim_{x\to 0} \frac{x^2\cos x}{x\sin x}$

2. (8pts) Consider the sequence with first five terms given by

$$a_1 = 3 \qquad a_2 = \frac{3}{4} \qquad a_3 = \frac{3}{16} \qquad a_4 = \frac{3}{64} \qquad a_5 = \frac{3}{256}$$

   Assuming the sequence continues on in this same manner, answer the following questions:

   (a) (2pts) Find a formula for $a_n$. $a_n =$ ____________

   (b) (2pts) $\lim_{n\to\infty} a_n =$ ____________.

   (c) (2pts) Is this sequence convergent or divergent? ____________

   (d) (2pts) $\sum_{n=1}^{\infty} a_n =$ ____________ (write 'diverges' if the series diverges)

3. (12pts) In this problem, we will compute the following improper integral in two steps.

$$\int_0^1 x \ln x \, dx$$

**Note:** This is an improper integral because the integrand $x \ln x$ is not continuous at $x = 0$.

(a) (6pts) First compute $\displaystyle\int_t^1 x \ln x \, dx$ **Note:** Your answer will be a function of $t$.

(b) (6pts) Compute the limit as $t \to 0^+$ of you answer from part (a) above. If this limit does not exist, write 'DNE'. **Note:** You will have to use L'Hôpital's rule for part of this computation.

4. (10pts) If the series is convergent, evaluate it. Otherwise, write 'divergent' and explain why.

(a) (5pts) $\displaystyle\sum_{n=1}^{\infty} 4\left(\frac{5^n}{3^{n-1}}\right)$

(b) (5pts) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{2}{n+2} - \frac{2}{n+3}\right)$

5. (20pts) Determine, by whatever method you wish, whether the following series are convergent or divergent. Circle 'C' if the series is convergent or 'D' if the series is divergent. You do not need to show work.

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} 6e^{-n}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \frac{3}{(2n)!}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \frac{5n+1}{n+3}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{n}$

**C** **D** $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{7}\right)^{n+5}$

**C** **D** $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

6. (10pts) Circle T if the statement is true and F if the statement is false.

**T** **F** If the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent, then the series $\sum_{n=1}^{\infty} a_n$ is convergent.

**T** **F** If $\{a_n\}_{n=1}^{\infty}$ is positive and decreases to zero, then $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent.

**T** **F** If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

**T** **F** If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n \neq 0$.

**T** **F** If both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge, then $\sum_{n=1}^{\infty} (a_n + b_n)$ converges.

7. (10pts) Find the first four terms of the power series representation

$$f(x) = \frac{1}{(2-x)^2} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

$c_0 =$ ________ $c_1 =$ ________ $c_2 =$ ________ $c_3 =$ ________

What is the radius of convergence of this power series? ____________

8. (10pts) Determine the **interval** of convergence of the series

$$\sum_{n=1}^{\infty} \frac{3(x-1)^n}{5^n n^4}$$