1220-90 Exam 2
Fall 2012

Name _____

Instructions. Show all work and include appropriate explanations when necessary. A correct answer without accompanying work may not receive full credit. Please try to do all all work in the space provided. Please circle your final answer.

1. (16pts) Evaluate the following limits using L'Hôpital's rule or any other method.

   (a) (4pts) $\lim_{x\to 0} \dfrac{x}{\ln(1-x)}$

   (b) (4pts) $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2}$

   (c) (4pts) $\lim_{x\to \infty} \dfrac{\ln x}{\sqrt{x}}$

   (d) (4pts) $\lim_{x\to \infty} x^2 e^{-x}$

2. (8pts) Consider the sequence with first five terms given by

$$a_1 = \frac{1}{3} \qquad a_2 = \frac{2}{4} \qquad a_3 = \frac{3}{5} \qquad a_4 = \frac{4}{6} \qquad a_5 = \frac{5}{7}$$

   (a) (3pts) Assuming the sequence continues on in this manner, find a formula for a_n. $a_n =$_____

   (b) (3pts) $\lim_{n\to\infty} a_n =$_____.

   (c) (2pts) Is this sequence convergent or divergent? _____

3. (11pts) In this problem, you will compute the value of an indefinite integral in two steps.

   (a) (7pts) Use substitution to compute the definite integral

$$\int_0^t xe^{-x^2}\,dx.$$

   Note: Your answer should be a function of t.

   (b) (4pts) Take the limit as $t \to \infty$ of your answer in part (a) to find

$$\int_0^\infty xe^{-x^2}\,dx = \lim_{t\to\infty}\int_0^t xe^{-x^2}\,dx.$$

4. (15pts) All of the following series are convergent. Evaluate them.

   (a) (5pts) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n$

   (b) (5pts) $\displaystyle\sum_{n=1}^{\infty}\frac{5}{2^{n-1}}$

   (c) (5pts) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

5. (20pts) Determine, by whatever method you wish, whether the following series are convergent or divergent. Circle 'C' if the series is convergent or 'D' if the series is divergent.

C **D** $\sum_{n=1}^{\infty}(-1)^n$

C **D** $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

C **D** $\sum_{n=1}^{\infty}\frac{1}{2n^3+3}$

C **D** $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$

C **D** $\sum_{n=1}^{\infty}ne^{-n^2}$

C **D** $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

C **D** $\sum_{n=1}^{\infty}\frac{n}{n+2}$

C **D** $\sum_{n=1}^{\infty}\frac{n}{3^n}$

C **D** $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$

C **D** $\sum_{n=1}^{\infty}\left(\frac{2}{\pi}\right)^{n-1}$

6. (10pts) Find the first four terms of the power series representation

$$f(x)=\frac{3}{(1+2x)}=c_0+c_1x+c_2x^2+c_3x^3+\ldots$$

c_0 =_____

c_1 =_____

c_2 =_____

c_3 =_____

What is the radius of convergence of this power series? _____

7. (10pts) Use the integral test to determine whether or not the series

$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$$

converges or diverges.

8. (10pts) Determine the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n2^n}$$