1. (25pts) Find the indicated derivatives. No need to simplify.

(a) (5pts) \( D_x (\cosh^3 x) \)

\[
= 3 \cosh^2 x \left( \sinh x \right)
\]

(b) (5pts) \( D_x (e^{\sqrt{x}}) = D_x \left( e^{x^{1/2}} \right) \)

\[
= e^{x^{1/2}} \cdot \frac{1}{2} x^{-1/2} = \frac{e^{\sqrt{x}}}{\sqrt{x}}
\]

(c) (5pts) \( D_x (\ln (x^3 + 5)) \)

\[
= \frac{3x^2}{x^3 + 5}
\]

(d) (5pts) \( D_x (\sin^{-1} (3x + 5)) \)

\[
= \frac{3}{\sqrt{1 - (3x+5)^2}}
\]

(e) (5pts) \( D_x (x \cos x) \)

\[
\[
= e^{\ln x \cos x} \left( \frac{\cos x}{x} - \ln x \sin x \right)
\]

\[
= x \cos x \left( \frac{\cos x}{x} - \ln x \sin x \right)
\]
2. (8pts) Let \( f(x) = x^3 + 2x + 1 \).

(a) (3pts) Why can we conclude that \( f(x) \) has an inverse? **Hint:** Conclude something from \( f'(x) \).

\[ f'(x) = 3x^2 + 2 \]

since derivative is always positive, \( f \) is 1-1.

(b) (2pts) Fill in the blank: \( f^{-1}(1) = \) \( \sqrt[3]{1} \).

(c) (3pts) Compute \((f^{-1})'(1)\), i.e. compute the derivative of the inverse function to \( f \) evaluated at 1.

\[ (f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(1)} = \frac{1}{3 \cdot 1^2 + 2} = \frac{1}{5} \]

3. (10pts) Evaluate the following. Any answer representing an angle should be given in radians.

(a) \( \cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6} \)

(b) \( \sin^{-1}(0) = 0 \)

(c) \( \sin(\sin^{-1}(1)) = \frac{1}{\sqrt{2}} \)

(d) \( \cos(\sin^{-1}(\frac{1}{2})) = \frac{1}{2} \)

(e) \( \sin(\tan^{-1}(\frac{\sqrt{3}}{3})) = \frac{\sqrt{3}}{3} \)

4. (10pts) A particular radioactive isotope decays from 10 grams to 7 grams in 32 hours. Let \( A(t) \) denote the amount (in grams) of the isotope present after \( t \) hours.

(a) (6pts) Find constants \( C \) and \( k \) such that \( A(t) = Ce^{kt} \).

\[ C = 10 \]

\[ 10 = A(0) = Ce^{0} = C \]

\[ k = \frac{\ln(\frac{7}{10})}{32} \]

\[ 7 = 10e^{k(32)} \Rightarrow \frac{7}{10} = e^{k(32)} \]

\[ \ln\left(\frac{7}{10}\right) = 32k \Rightarrow k = \frac{\ln\left(\frac{7}{10}\right)}{32} \]

(b) (4pts) What is the half-life of this isotope? In other words, how long will it take before half of the isotope has decayed? No need to simplify.

\[ t = \frac{32 \cdot \ln\left(\frac{1}{2}\right)}{\ln\left(\frac{7}{10}\right)} \]

\[ t = \frac{32 \cdot \ln\left(\frac{1}{2}\right)}{\ln\left(\frac{7}{10}\right)} \]
5. (25pts) Evaluate the following indefinite integrals. Remember $+C$!!

(a) $\int \sin^3 x \, dx$

$\int \sin^2 x \sin x \, dx = \int (1 - \cos^2 x)(\sin x) \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$= - \int (1 - u^2) \, du = - u + \frac{1}{3} u^3 + C$

$= -\cos x + \frac{1}{3} \cos^3 x + C$

(b) $\int xe^{3x} \, dx$

$u = x \quad du = dx$

$dv = e^{3x} \, dx \quad v = \frac{1}{3} e^{3x}$

$= \frac{1}{3} xe^{3x} - \frac{1}{3} \int e^{3x} \, dx$

$= \frac{1}{3} xe^{3x} - \frac{1}{9} e^{3x} + C$

(c) $\int \frac{e^x}{1 + 9e^{2x}} \, dx$

$u = 3e^x \quad du = 3e^x \, dx$

$= \frac{1}{3} \int \frac{du}{1 + u^2}$

$= \frac{1}{3} \tan^{-1} u + C$

$= \frac{1}{3} \tan^{-1} (3e^x) + C$

(d) $\int \sin^2 x \cos^2 x \, dx$

$= \frac{1}{4} \int (1 - \cos (2x))(1 + \cos (2x)) \, dx$

$= \frac{1}{4} \int (1 - \cos^2 (2x)) \, dx$

$= \frac{1}{4} \int (1 - \frac{1}{2}(1 + \cos (4x))) \, dx$

$= \frac{1}{4} \int (-\frac{1}{2} \cos (4x)) \, dx$

$= \frac{1}{8} x - \frac{1}{32} \sin (4x) + C$

(e) $\int x \ln (2x) \, dx$

$u = \ln (2x) \quad du = \frac{1}{x} \, dx$

$dv = x \, dx \quad v = \frac{x^2}{2}$

$= \frac{x^2}{2} \ln (2x) - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \ln (2x) - \frac{x^2}{4} + C$
6. (12pts) Evaluate the following indefinite integral using rationalizing substitution in three steps.

\[ \int \frac{1}{x^2 \sqrt{x^2 + 1}} \, dx \]

(a) (4pts) First, write as an integral with respect to \( t \) by using the rationalizing substitution \( x = \tan t \).

\[ x = \tan t \Rightarrow dx = \sec^2 t \, dt \quad \Rightarrow \quad \sqrt{x^2 + 1} = \sqrt{\tan^2 t + 1} = \sec t. \]

\[ \int \frac{dx}{x^2 \sqrt{x^2 + 1}} = \int \frac{\sec^2 t \, dt}{\tan^2 t \cdot \sec t} = \int \frac{\sec t \, dt}{\tan^2 t} = \int \frac{\cos t}{\sin^2 t} \, dt \]

(b) (4pts) Second, evaluate the integral from part (a). You answer should be in terms of \( t \).

\[ \int \frac{\cos t}{\sin^2 t} \, dt = \int \frac{du}{u^2} = \int u^{-2} \, du = -u^{-1} + C = -\frac{1}{\sin t} + C \]

(c) (4pts) Finally, write you answer to part (b) in terms of \( x \).

\[ x = \tan t \Rightarrow \tan t = \frac{x}{1} \]

\[ \sqrt{x^2 + 1} \quad \Rightarrow \quad \sin t = \frac{x}{\sqrt{x^2 + 1}} \]

\[ \frac{-1}{\sin t} + C = \frac{-\sqrt{x^2 + 1}}{x} + C \]

7. (10pts) Use partial fractions to find \( \int \frac{2}{x^2 - 3x + 2} \, dx \)

\[ \frac{2}{(x^2 - 3x + 2)} = \frac{A}{(x - 2)} + \frac{B}{(x - 1)} = \frac{A(x - 1) + B(x - 2)}{(x - 2)(x - 1)} \quad \Rightarrow \quad 2 = A(x - 1) + B(x - 2). \]

Plug in \( x = 1 \):

\[ 2 = -B \quad \Rightarrow \quad B = -2. \]

Plug in \( x = 2 \):

\[ 2 = A \quad \Rightarrow \quad A = 2. \]

\[ \int \frac{2}{x^2 - 3x + 2} \, dx = \int \left( \frac{2}{x - 2} - \frac{2}{x - 1} \right) \, dx = 2 \ln|x - 2| - 2 \ln|x - 1| + C \]