

1220-90 Final Exam
Summer 2013

Name _____

Instructions. Show all work and include appropriate explanations when necessary. Answers unaccompanied by work may not receive credit. Please try to do all work in the space provided and circle your final answers. The last page contains some useful formulas.

1. (10pts) Compute the following derivatives:

(a) (5pts) $D_x(\tan^{-1}(x^2))$

(b) (5pts) $D_x(x^{7x})$

2. (15pts) Compute the following indefinite integrals: **Remember:** $+C!$

(a) (5pts) $\int \frac{1}{4x+3} dx$

(b) (5pts) $\int \cos^3 x dx$

(c) (5pts) $\int x^2 \ln x dx$

3. (5pts) a formula for the inverse of the function

$$f(x) = \ln(3x + 8)$$

4. (10pts) Use the rationalizing substitution $x = \tan t$ to find the antiderivative

$$\int \frac{1}{(x^2 + 1)^{3/2}} dx$$

5. (10pts) Find the antiderivative $\int \frac{x}{x^2 - 3x + 2} dx$.

6. (14pts) Determine, by whatever method you wish, whether the following series are convergent or divergent. Circle 'C' if the series is convergent or 'D' if the series is divergent. No work is necessary.

C D $\sum_{n=1}^{\infty} \frac{1}{\ln n}$

C D $\sum_{n=1}^{\infty} \frac{4n}{3n+9}$

C D $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

C D $\sum_{n=1}^{\infty} e^{-n}$

C D $\sum_{n=1}^{\infty} \frac{6^n}{n!}$

C D $\sum_{n=1}^{\infty} \left(\frac{7}{6}\right)^{n-1}$

C D $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^2+1}}$

7. (10pts) Use the integral test to determine whether the series

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$$

converges or diverges.

8. (10pts) Find the **interval** of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}(-5)^n}$$

9. (12pts) Let

$$f(x) = \ln(1 + x + x^2)$$

(a) (6pts) Find the following derivatives of $f(x)$ evaluated at $x = 0$:

$$f(0) = \underline{\hspace{2cm}}$$

$$f'(0) = \underline{\hspace{2cm}}$$

$$f''(0) = \underline{\hspace{2cm}}$$

(b) (3pts) Use your computations above to write out $P_2(x)$, the second degree MacLaurin polynomial for $f(x)$.

(c) (3pts) Use $P_2(x)$ to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\ln(1 + x + x^2) - x}{x^2}.$$

10. (12pts) Match the equation with the type of conic section it describes by writing the letter in the blank provided.

_____ $x + y^2 - 2y = 1$

A. circle

_____ $x^2 + y^2 - 2y = -1$

B. ellipse

_____ $x^2 + y^2 - 2y = -3$

C. parabola

_____ $x^2 + y^2 - 2y = 0$

D. hyperbola

_____ $2x^2 + y^2 - 2y = 1$

E. a point

_____ $x^2 - y^2 - 2y = 0$

F. the empty set (no solution)

11. (a) (4pts) Find the polar coordinates of the point with Cartesian coordinates $(\sqrt{2}, \sqrt{2})$.

(b) (4pts) Find the Cartesian coordinates of the point with polar coordinates $(4, \frac{2\pi}{3})$.

12. (14pts) Match the polar equation to the type of curve it determines by writing the letter in the blank provided. Every answer will be used exactly once.

_____ $r = \frac{6}{\sin \theta}$

A. a circle centered at the origin

_____ $r = e^{-\theta}$

B. a circle centered on the y-axis

_____ $r = \sec \theta$

C. a horizontal line

_____ $r = 8 \sin \theta$

D. an angled line

_____ $r = \pi$

E. a vertical line

_____ $r = \frac{1}{\sin \theta - \cos \theta}$

F. a circle centered on the x-axis

_____ $r = 4 \cos \theta$

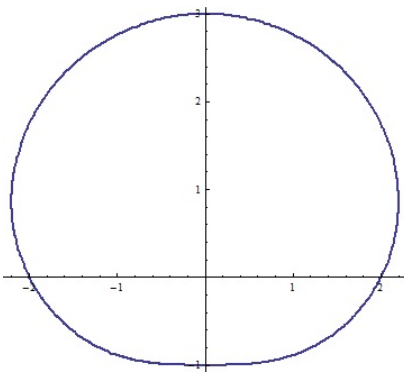
G. a spiral

13. (20pts) Consider the limaçon determined by the polar equation $r = 2 + \sin \theta$. A graph of this polar curve is given at the bottom of the page.

(a) (6pts) Find the slope of the tangent line to the curve at the point $(2, 0)$ (that is, when $\theta = 0$).

(b) (9pts) Find the area of the region inside the limaçon.

(c) (5pts) Set up an integral to find the perimeter of the limaçon. Do not attempt to solve.



Trigonometric Formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

Inverse Trigonometric Formulas:

$$D_x \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad -1 < x < 1$$

$$D_x \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \quad -1 < x < 1$$

$$D_x \tan^{-1} x = \frac{1}{1+x^2}$$

$$D_x \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \quad |x| > 1$$

Calculus with Polar Curve $r = f(\theta)$:

$$A = \int_a^b \frac{1}{2} [f(\theta)]^2 d\theta$$

$$L = \int_a^b \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta$$

$$m = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$