Calculus I
Practice Problems 8

1. Let $y = (x-2)^2 + \frac{1}{3}(x-1)^3$. Find the intervals in which the function is increasing and decreasing, and where it is concave up and concave down. Sketch the graph.

2. Consider the function $y = 2x^3 + 3x^2 - 12x + 11$. Find the intervals in which the function is concave up, and in which it is concave down. Sketch the graph.

3. For the following function, find a) all critical values, b) intervals in which the function is increasing and where it is decreasing, c) intervals in which the function is concave up or concave down:

$$g(x) = x^4 - 4x^3 + 4x^2 + 2 \, .$$

Sketch the graph.

4. Consider the function

$$y = \frac{1}{x} - \frac{5}{2x^2} + \frac{4}{3x^3} \, ,$$

defined for $x > 0$.
a) Find the intervals in which the function is increasing, and the intervals in which it is decreasing.
b) Find the intervals in which the function is concave up, concave down. Sketch the graph.

5. Consider the function

$$f(x) = \sin x + (\cos x)^2$$

as defined on the interval $[-\pi, \pi]$. Find a) all critical values, b) all points of inflection, c) the value at which the function takes its maximum. Sketch the graph.

6. Let

$$y = \frac{1}{x^2 + 1}.$$

Find the x coordinate of the points of inflection of the curve so defined. Sketch the graph.

In each of the following, derive enough information about the curve to sketch the graph, showing clearly all asymptotes.

7. $y = \dfrac{x}{x^2 + 1}$.

8. $y = \dfrac{x}{(x-1)(x-2)}$.

9. $y = \dfrac{x^2}{(x-1)(x-2)}$.

10. $y = \sqrt{1 - \cos x} \quad -2\pi \leq x \leq 2\pi$.