Calculus I
Practice Problems 11

1. A solid is formed over the region in the first quadrant bounded by the curve $y = \sqrt{10 - x}$ so that the section by any plane perpendicular to the x-axis is a semicircle. What is the volume of this solid?

2. A solid is formed over the region in the first quadrant bounded by the curve $y = \sqrt{4 - x}$ so that the section by any plane perpendicular to the x-axis is a square. What is the volume of this solid?

3. A solid is formed over the region in the first quadrant bounded by the curve $y = 2x - x^2$ so that the section by any plane perpendicular to the x-axis is a semicircle. What is the volume of this solid?

4. The region in the first quadrant bounded by $y = \sqrt{x^2 - 1},\ y = 0,\ x = 1,\ x = 4$ is revolved around the x-axis. Find the volume of the resulting solid.

5. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = x^2,\ x = 2$ and the x-axis.

6. The region in the first quadrant under the curve $y^2 = 2x - x^2$ is rotated about the y-axis. Find the volume of the resulting solid.

7. The region in the first quadrant bounded by $y = x^4$ and $x = 1$ is revolved around the y-axis. Find the volume of the resulting solid.

8. The region in the first quadrant bounded by $y = x - x^2$ and $y = x - x^3$ is revolved around the x-axis. Find the volume of the resulting solid.

9. The *average value* of a function $y = f(x)$ defined over an interval $[a, b]$ is defined to be

$$y_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)dx\ .$$

(If, for example, the graph of $y = f(x)$ were a histogram of the grades on an exam, then y_{ave} would be the average grade.) Find the average of $y = \sin x$ over the interval $[0, \pi]$.

10. Let $g(x) = x^2 + x^3$ for x in the interval $0 \le x \le 10$. Find the average, or mean, value of g on the interval. Find the average slope of the graph of $y = g(x)$ on the interval.