Calculus I
Practice Problems 1: Answers

1. Find the equation of the line which goes through the point (2,-1) and is parallel to the line given by the equation \(2x - y = 1\).

**Answer.** The given line has the equation \(y = 2x - 1\); our line, being parallel will have equation \(y = 2x + b\) for some \(b\). Substitute \(x = 2\), \(y = -1\): \(-1 = 2(2) + b\), so \(b = -5\). The equation is \(y = 2x - 5\).

2. Find the equation of the line which goes through the point (2,-1) and is perpendicular to the line given by the equation \(2x - y = 1\).

**Answer.** The given equation: \(y = 2x - 1\) has slope 2. Thus the line we seek has slope \(-1/2\). Its equation is

\[
\frac{y - (-1)}{x - 2} = \frac{-1}{2} \quad \text{or} \quad y = \frac{-1}{2} x.
\]

3. Find the equation of the line which goes through the point (1,2) and is parallel to the line through the points (0,1) and (-2,7).

**Answer.** The line through the points (0,1) and (-2,7) has slope

\[
\frac{7 - 1}{-2 - 0} = -3.
\]

So, our line through (1,2) and slope \(m = -3\) has the equation

\[
\frac{y - 2}{x - 1} = -3
\]

\[
y - 2 = -3(x - 1) = -3x + 3
\]

\[
y = -3x + 5
\]

4. Find the derivative: \(f(x) = x^3 - x^2 + 1\)

**Answer.** \(3x^2 - 2x\).

5. Find the derivative: \(f(x) = x^5 + 3x^4 - 2x^2 + 4x - 7\)

**Answer.** \(5x^4 + 12x^3 - 4x + 4\).

6. Find the derivative: \(f(x) = x^4 - 2x^3 + 5x^2 - x + 7\)

**Answer.** \(4x^3 - 6x^2 + 10x - 1\).

7. Find the derivative: \(f(x) = 3x^{-1} + x^3\)
8. Find the equation of the tangent line to the graph of \( y = x^3 - 3x^2 + x \) at the point \((2,-2)\).

**Answer.** Differentiating, we find \( dy/dx = 3x^2 - 6x + 1 \). Evaluate at \( x = 2 \): \( dy/dx = 3(2)^2 - 6(2) + 1 = 1 \). This is the slope of the tangent line at \((2,-2)\), so its equation is

\[
\frac{y - (-2)}{x - 2} = 1 \quad \text{or} \quad y = x - 4.
\]

9. Let \( y = 16x^{-1} - x^2 \). At what point(s) is the tangent line horizontal?

**Answer.** Differentiating, \( dy/dx = -16x^{-2} - 2x \). The tangent line is horizontal when its slope is zero. So, we solve

\[
\frac{-16}{x^2} - 2 = 0 \quad \text{or} \quad \frac{-16 - 2x^3}{x^2} = 0,
\]
which has the solution \( x = -2 \). For this value of \( x \), \( y = 16(-2)^{-1} - 2^2 = -8 - 4 = -12 \). The answer is \((-2,-12)\).

10. Let \( y = 4x^4 + x \). At what point is the tangent line to the graph perpendicular to the line tangent to the graph at \((0,0)\)?

**Answer.** Differentiating, \( dy/dx = 16x^3 + 1 \). At \( x = 0 \), we get \( dy/dx = 1 \); this is the slope of the tangent line at \((0,0)\). A line perpendicular to this line has slope -1, so we must solve \( dy/dx = -1 \), or

\[
16x^3 + 1 = -1, \quad 16x^3 = -2,
\]
which has the solution \( x = -1/2 \). For this value of \( x \), \( y = 4(-1/2)^4 + 1 = 5/4 \). Thus the point is \((-1/2,5/4)\).

11. Find the derivative: \( f(x) = \left( x^2 + \frac{1}{x^4} \right) (x^3 - x^2 + 1) \)

**Answer.** \( (x^2 + x^{-3}) (3x^2 - 2x) + (2x - 3x^{-3}) (x^3 - x^2 + 1) = 5x^4 - 4x^3 + 2x + x^{-2} - 3x^{-4} \).

12. Find \( f' \) and \( f'' \): \( f(x) = \left( x + \frac{1}{x} \right) (x^2 + 1) \)

**Answer.** Use the product rule:

\[
f'(x) = (x + x^{-1}) (2x) + (1 - x^{-2}) (x^2 + 1)
= 2x^2 + 2 + x^2 + 1 - x^{-2} = 3x^2 - x^{-2} + 2 .
\]

Differentiating again,

\[
f''(x) = 6x + 2x^{-3}.
\]