Solutions for Introduction to Polynomial Calculus

Section 4 Problems - Antiderivatives of Polynomials

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Calling the function in each problem \( f(x) \) and using the three antidifferentiation rules corresponding to the previous three differentiation rules:

The antiderivative of \( f(x) = x^n \) is \( \int f(x)\,dx = \frac{x^{n+1}}{n+1} + C \).

If \( f(x) = u(x) + v(x) \) then \( \int f(x)\,dx = \int u(x)\,dx + \int v(x)\,dx \).

If \( f(x) = c(u(x)) \) where \( c \) is a constant, then \( \int f(x)\,dx = c \int u(x)\,dx \).

(1) \( \int f(x)\,dx = x^2 - 3x + C \). You should check this by taking its derivative!

(2) \( \int f(x)\,dx = x^3 - 2x^2 + 5x + C \).

(3) \( \int f(x)\,dx = \frac{x^6}{6} + \frac{x^4}{2} + x + C \).

(4) \( \int f(x)\,dx = x^{10} - 4x^2 + C \).

Find the general antiderivative then impose the condition to determine \( C \):

(5) \( F(x) = \int f(x)\,dx = \frac{x^3}{3} - 5x + C \) and \( F(0) = 2 \) says \( C = 2 \), so \( F(x) = \frac{x^3}{3} - 5x + 2 \).

(6) \( F(x) = \int f(x)\,dx = 2x^4 - x^2 + C \) and \( F(1) = 4 \) says \( 2 - 1 + C = 4 \), so \( C = 3 \) and \( F(x) = 2x^4 - x^2 + 3 \).

(7) \( F(x) = \int f(x)\,dx = \frac{x^4}{2} + C \) and \( F(1) = 1 \) says \( \frac{1}{2} + C = 1 \), so \( C = \frac{1}{2} \) and \( F(x) = \frac{x^4}{2} + \frac{1}{2} \).

(8) \( F(x) = \int f(x)\,dx = \frac{x^4}{4} - \frac{x^2}{2} + C \) and \( F(2) = 1 \) says \( 4 - 2 + C = 1 \), so \( C = -1 \) and \( F(x) = \frac{x^4}{4} - \frac{x^2}{2} - 1 \).

(9) The derivative of velocity is acceleration, and the acceleration of any body near the earth’s surface under only the force of gravity is \( -32 \) feet per second squared. Since the (vertical) velocity is then the antiderivative of the acceleration,

\[ v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C \]

feet per second. We are given that \( v(0) = 64 \) feet per second, so \( 0 + C = 64 \) and \( v(t) = -32t + 64 \) feet per second is the velocity after \( t \) seconds. The ball will achieve its maximum height when its vertical velocity changes from positive to negative, i.e., when \( v(t) = -32t + 64 = 0 \), so when \( t = 2 \) seconds.

(10) The derivative of (vertical) displacement, or height, is velocity, and the velocity of the ball is \( v(t) = -32t + 64 \) from the previous problem. Since the (vertical) displacement is then the antiderivative of the velocity,

\[ s(t) = \int v(t)\,dt = \int -32t + 64\,dt = -16t^2 + 64t + C \]
feet. We are given that \( s(0) = 6 \) feet, so \( 0 + 0 + C = 6 \) and \( s(t) = -16t^2 + 64t + 6 \) feet is the height of the ball after \( t \) seconds. Since the ball achieves its maximum height when \( t = 2 \) seconds, the maximum height it achieves is \( s(2) = 70 \) feet.