Solutions for Introduction to Polynomial Calculus

Section 3 Problems - The Derivative of a Polynomial

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Calling the function in each problem \( f(x) \) and using the three rules from the previous section:

The derivative of \( f(x) = x^n \) is \( f'(x) = nx^{n-1} \).

If \( f(x) = u(x) + v(x) \) then \( f'(x) = u'(x) + v'(x) \).

If \( f(x) = c(u(x)) \) where \( c \) is a constant, then \( f'(x) = c(u'(x)) \).

(1) \( f'(x) = 9x^8 \).
(2) \( f'(x) = 100x^{49} \).
(3) \( f'(x) = 3 \).
(4) \( f'(x) = 3x^2 - 2 \).
(5) \( f'(x) = 8x^3 + 3x^2 - 10x + 1 \).
(6) \( f'(x) = 11x^{10} - 18x^8 + 15 \).

Computing \( f'(x) \) and setting \( x \) equal to the \( x \) value at the given point on the graph:

(7) \( f'(x) = 3x^2 \), and \( f'(1) = 3 \) gives the slope of the curve at \((1,1)\), as in problem (7) of the previous section. If you prefer when the function is given as \( y = f(x) \) you may prefer to use \( \frac{dy}{dx} \) (Leibniz notation) instead of \( f'(x) \) (Newton notation). Then instead of \( f'(1) \) we sometimes write \( \frac{dy}{dx}|_{x=1} \) or even \( \frac{dy}{dx}(1) \).

(8) \( f'(x) = 2x \), and \( f'(0) = 0 \) gives the slope of the curve at \((0,0)\), as in problem (2) of the previous section.

(9) \( f'(x) = 3x^2 - 2x \), and \( f'(1) = 1 \) gives the slope of the curve at \((1,0)\).

(10) \( f'(x) = 4x^3 - 6x^2 + 5 \), and \( f'(2) = 13 \) gives the slope of the curve at \((2,7)\). The \( y \)-value comes from evaluating \( f(2) \). The equation for the tangent line is \( y - 7 = 13(x - 2) \).

(11) \( f'(x) = 10x^9 - 5x^4 \), and \( f'(1) = 5 \) gives the slope of the curve at \((1,0)\). The \( y \)-value comes from evaluating \( f(1) \). The equation for the tangent line is \( y - 0 = 5(x - 1) \).

(12) \( f'(x) = 2x - 2 \), and \( f'(x) = 0 \) when \( 2x - 2 = 0 \) or \( x = 1 \), \( f'(x) > 0 \) when \( 2x - 2 > 0 \) or \( x > 1 \), and \( f'(x) < 0 \) when \( 2x - 2 < 0 \) or \( x < 1 \). In words, the curve has positive slope for \( x > 1 \), negative slope for \( x < 1 \) and zero slope for \( x = 1 \).

(13) The (vertical) velocity of the ball \( t \) seconds after it is thrown is given by \( \frac{ds}{dt} = s'(t) = -32t + 32 \). The ball reaches its maximum height when its velocity changes from positive to negative, i.e., when \( s'(t) = -32t + 32 = 0 \) or \( t = 1 \). The height of the ball at \( t = 1 \) is \( s(1) = 22 \) feet.

(14) The (vertical) acceleration of the ball \( t \) seconds after it is thrown is given by \( \frac{d^2s}{dt^2} = s''(t) = -32 \) feet per second per second or feet per second squared. The velocity loses a constant 32 feet per second upward every second.