Solutions for Introduction to Polynomial Calculus

Section 3 Problems - The Derivative of a Polynomial

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Calling the function in each problem $f(x)$ and using the three rules from the previous section:

The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$.

If $f(x) = u(x) + v(x)$ then $f'(x) = u'(x) + v'(x)$.

If $f(x) = c(u(x))$ where c is a constant, then $f'(x) = c(u'(x))$.

(1) $f'(x) = 9x^8$.

(2) $f'(x) = 100x^{49}$.

(3) $f'(x) = 3$.

(4) $f'(x) = 3x^2 - 2$.

(5) $f'(x) = 8x^3 + 3x^2 - 10x + 1$.

(6) $f'(x) = 11x^{10} - 18x^8 + 15$.

Computing $f'(x)$ and setting x equal to the x value at the given point on the graph:

(7) $f'(x) = 3x^2$, and $f'(1) = 3$ gives the slope of the curve at $(1, 1)$, as in problem (7) of the previous section. If you prefer when the function is given as $y = f(x)$ you may prefer to use $\frac{dy}{dx}$ (Leibniz notation) instead of $f'(x)$ (Newton notation). Then instead of $f'(1)$ we sometimes write $\frac{dy}{dx}|_{x=1}$ or even $\frac{dy}{dx}(1)$.

(8) $f'(x) = 2x$, and $f'(0) = 0$ gives the slope of the curve at $(0, 0)$, as in problem (2) of the previous section.

(9) $f'(x) = 3x^2 - 2x$, and $f'(1) = 1$ gives the slope of the curve at $(1, 0)$.

(10) $f'(x) = 4x^3 - 6x^2 + 5$, and $f'(-1) = -5$ gives the slope of the curve at $(-1, 1)$. The y-value comes from evaluating $f(-1)$.

(11) $f'(x) = 100x^{49} - 100x$, and $f'(1) = 0$ gives the slope of the curve at $(1, -48)$. The y-value comes from evaluating $f(1)$.

(12) $f'(x) = 2x - 2$, and $f'(x) = 0$ when $2x - 2 = 0$ or $x = 1$, $f'(x) > 0$ when $2x - 2 > 0$ or $x > 1$, and $f'(x) < 0$ when $2x - 2 < 0$ or $x < 1$. In words, the curve has positive slope for $x > 1$, negative slope for $x < 1$ and zero slope for $x = 1$.

(13) The (vertical) velocity of the ball t seconds after it is thrown is given by $\frac{ds}{dt} = s'(t) = -32t + 32$. The ball reaches its maximum height when its velocity changes from positive to negative, i.e., when $s'(t) = -32t + 32 = 0$ or $t = 1$. The height of the ball at $t = 1$ is $s(1) = 22$ feet.

(14) The (vertical) acceleration of the ball t seconds after it is thrown is given by $\frac{d^2s}{dt^2} = s'(t) = -32$ feet per second per second or feet per second squared. The velocity loses a constant 32 feet per second upward every second.