MATH 1210 - 9 Exam #2

Problem 1

(a) $\lim_{h\to 0} \frac{(5+h)^2-25}{h} = \lim_{h\to 0} \frac{25+10h+h^2-25}{h}$ (5pts)

$= \lim_{h\to 0} \frac{h(10+h)}{h} = \lim_{h\to 0} (10+h) = \boxed{10}$ (5pts)

(b) $\lim_{x\to 4} \frac{\frac{2}{x}-\frac{2}{4}}{x-4} = \lim_{x\to 4} \frac{\frac{2(4-x)}{4x}}{x-4}$

(5pts) $= \lim_{x\to 4} \frac{2(4-x)}{4x} \cdot \frac{1}{(x-4)} = \lim_{x\to 4} \left(-\frac{2}{4x}\right)$

$= \boxed{-\frac{1}{8}}$

Problem 2

Method 1 : $f(x) = (x+6)(x^2-4)$

(product rule)

$f'(x) = 1 \cdot (x^2-4) + (x+6) \cdot (2x)$ (5pts)

$= 3x^2 + 12x - 4$ (5pts)

$f''(x) = 6x + 12$ (5pts)

$f''(x) = 0 \Rightarrow 6x + 12 = 0 \Rightarrow \boxed{x = -2}$ (5pts)

Method 2: (expand)

$f(x) = x^3 + 6x^2 - 4x - 24$ (5pts)

$f'(x) = 3x^2 + 12x - 4$ (5pts)

$f''(x) = 6x + 12$ (5pts)

Let $f''(x) = 0 \Rightarrow 6x + 12 = 0 \Rightarrow \boxed{x = -2}$ (5pts)

Problem 3

$$f(x) = (\cos 8x + 9)^{10}$$

Chain Rule)

$$f'(x) = \underbrace{10(\cos 8x + 9)^9}_{(5\text{pts})} \cdot \underbrace{(\cos 8x + 9)'}_{(5\text{pts})}$$

$$= 10(\cos 8x + 9)^9 \cdot \underbrace{(-\sin 8x)}_{(5\text{pts})} \cdot \underbrace{(8)}_{(5\text{pts})}$$

$$= -80 \sin 8x (\cos 8x + 9)^9$$

OR Let $u = \cos 8x + 9$ (5pts) $\frac{du}{dx} = (-\sin 8x)(8)$ (5 pts)

$$f'(x) = 10u^9 \cdot \frac{du}{dx} \quad \text{(5pts)}$$

$$= 10(\cos 8x + 9)^9 \cdot (-8 \sin 8x)$$

$$= -80 \sin 8x (\cos 8x + 9)^9 \quad \text{(5pts)}$$

Problem 4

$f(x) = \sqrt{x}$ (5pts)

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ (5pts)

when Δx is small,

$f(x+\Delta x) \approx f(x) + \Delta x \cdot f'(x)$

So $\sqrt{4.1} = \sqrt{4 + 0.1} \approx \sqrt{4} + (0.1) \cdot \frac{1}{2\sqrt{4}} \approx \boxed{2.025}$

(x = 4, Δx = 0.1)

(5pts)

$\sqrt{8.8} = \sqrt{9 - 0.2} \approx \sqrt{9} + (-0.2) \cdot \frac{1}{2\sqrt{9}} \approx \boxed{2.9667}$

(x = 9, Δx = −0.2)

(5pts)

Problem 5

$$y^3 - xy^2 + \cos xy = 2$$

opts $3y^2 \frac{dy}{dx} - \left(y^2 + 2xy\frac{dy}{dx}\right) - (\sin xy)\cdot\left(y + x\frac{dy}{dx}\right) = 0$

$3y^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} - (\sin xy)\cdot y - (\sin xy)\cdot x\frac{dy}{dx} =$

plug in $x=0$, $y=1$ and solve for $\frac{dy}{dx}$

$$3\frac{dy}{dx} - 1 = 0$$

$$\frac{dy}{dx} = \frac{1}{3} \quad \text{or} \quad y' = \frac{1}{3} \text{ @ } (0,1)$$

5pts

(0,1)

tangent line : $y - 1 = \frac{1}{3}(x - 0)$

$$\boxed{y = \frac{1}{3}x + 1}$$ (5pts).