Calculus I
Practice Exam 2, Summer 2002

1. Find the equation of the tangent line to the ellipse

$$x^2 + 2y^2 = 9$$

at the point (1,-2).

2. Given the relation

$$x^3 - y^2 + x + 2y = 7$$

between the variables x and y, express the derivative dy/dx as a function of x and y.

3. A curve C in the plane is the graph of the relation $\sqrt{y} = \sin(x/2)$.
a) Find the slope of the tangent line to the curve at the point $(\pi/2, 1/2)$.
b) Find a point in the first quadrant where the tangent line is horizontal.

4. Find those points on the curve $x^2 - 2xy + 3y^2 = 294$ at which the tangent line is vertical.

5. At a certain moment the hare and the mouse start out from the same point; the hare moving east at 8 mph, and the mouse north at 5 mph. After 3 hours, at what rate is the distance between them increasing?

6. At a race, I am sitting along the finish line, exactly 100 feet away from the track. The lead car is travelling at 560 feet/sec. Asuming I am watching the lead car, at what rate is my angle of vision changing as it crosses the finish line?

7. Water is pouring into a cone-shaped container (with vertex pointing down) at a rate of .05 cubic feet per minute. The cone is 8 feet deep and has a 5 foot diameter at the top. How fast is the water level rising when it is 3 ft. high?

8. Let $y = 10 + 7x + 2x^2 - x^3$. For what value of x is y a local maximum? a local minimum?

9. I want to build a rectangular building on a square base which has 60000 ft^3 of space. My painter will charge $\$h/2$ per ft^2 to paint a building h feet high, and will paint the roof at a cost of \$30 per ft^2. How high should the building be to minimize the cost of painting?

10. I have an elliptical meadow whose major axis is 100 meters and whose minor axis is 60 meters. I want to put a rectangular soccer field in the meadow, two of whose sides are parallel to the major axis, What are the dimensions of the field which maximize its area?

11. Let $y = (x^2 - 3)(x^2 - 5)$. Sketch the graph, clearly showing all local maxima and minima, points of inflection and concavity.

12. Graph $= \dfrac{x^3}{x^2 - 1}$

13. Graph $y = \dfrac{x^2}{x^2 - 1}$

14. Graph $y = \dfrac{x}{x^2 - 1}$