Mathematics 1210-90 Final Examination. July 29,30, 2003

You may use calculators and Tables of Integrals. You **MUST** show enough work to convince me that you know how to do the problems.

1. Find the derivatives of the following functions:

a) $\quad f(x) = (x^2 - 3)^2(2x + 4)$

b) $$g(x) = \frac{\sin x}{\cos x + 1}$$

2 a) Integrate : $$\int (6x^2 + 1)^5 x dx$$

b) Integrate : $$\int_1^4 \frac{1}{\sqrt{y}(\sqrt{y} + 1)^2} dy$$

3. Find the equation of the tangent line to the curve $2x^2 + y^2 - 2xy = 17$ at the point (1,5).

4. Consider the region in the first quadrant bounded by the curve $y = 12 - \frac{3}{4}x^2$. What are the dimensions of the largest rectangle with sides parallel to the coordinate axes which can be inscribed inside this region?

5. Sketch the graph of the function $y = 3x^4 - 4x^3 - 12x^2 + 2$. Find the x values of all local minima, maxima and points of inflection.

6. Find the solution to the differential equation

$$\frac{dy}{dx} = y^2 x^2 + y^2$$

such that $y(1) = 2$.

7. Find the area between the curve

$$y = \frac{x + 1}{x^3}$$

and the x-axis, as x ranges from 1 to 4.

8. The region between the curves $y = 8x$ and $y = x^4$ is rotated about the y-axis. Find the volume of the resulting solid.

9. Consider a curve given parametrically by $x = 4\cos^2 t,\ y = 3\sin^2 t$. Find the length of the piece of this curve running from $t = 0$ to $t = \pi/2$.

10. FInd the center of mass of the homogeneous region in the first quadrant bounded by the curve $x^4 + y = 1$.