**Calculus I**
**Final Exam, Spring 2003**

1. Find the derivatives of the following functions:

a) $f(x) = \dfrac{\sin x}{\sin x + 1}$

b) $g(x) = x\sqrt{1+x^2}$

2. Integrate:

a) $\displaystyle\int x\sqrt{1+x^2}dx =$

b) $\displaystyle\int \tan x \sec^2 x dx =$

3. A trough 12 feet long has a triangular cross section. The triangle is 4 feet across at the top and 3 feet deep. Water is pouring into the trough at the rate of 4 $ft^3$/min. At what rate is the height of the water rising when it is 2 feet above the vertex of the triangle?

4. A plane heading north at 480 mph passes over Salt Lake City at 1:00pm. Another plane, heading east at 360 mph passes over Salt Lake City at 1:15 pm. At what time are the planes closest to each other?

5. Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{3x+1}{y}$$

such that $y(0) = 4$.

6. Graph $y = \dfrac{x}{x^2-1}$, showing clearly all asymptotes and local maxima and minima.

7. Find the area of the region in the first quadrant bounded by the curve $y = x^3 - 9x^5$.

8. Find the volume of the solid obtained by rotating the curve $y = (x+1)^{-1},\ 0 \le x \le 9$ about the $x$-axis.

9.The triangle with vertices at (0,0), (1,0), (0,3) is filled with an inhomogeneous muck whose density at the point $(x,y)$ is $\delta(x,y) = 1 + x$. What is the mass of the muck?

10. A cylindrical water tank, 8 ft. in diameter and 12 feet high has its base 50 feet above ground level. How much work is done in filling the tank with water? (The density of water is 62.5 lbs/$ft^3$).