Mathematics 1210-90
Final Examination December 10,11, 2003

You must show your work. Just entering an answer will earn no points.

1. Let $f(x) = \dfrac{x^2}{1+x^2}$

a) $f'(x) =$

b) $f''(x) =$

2. Integrate:

a) $\displaystyle\int (4x^2 + x - x^{-2})dx =$

b) $\displaystyle\int \frac{\sqrt{x}}{(1+x^{3/2})^2}dx =$

3. The volume of a cone of radius r and height h is $V = \frac{\pi}{3}r^2h$. Water is pouring into a conical cup of radius 8 cm and height 10 cm at the rate of 120 cm/min. At what rate is the height of water in the cup rising when it is at $h = 5$ cm (and $r = 4$)?

4 *a*). Graph $y = 2x + \dfrac{1}{x}$ for $x > 0$.

b) What is the minimum value of y?

5. Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{3x}{y+1}$$

such that $y(0) = 4$.

6. A curve in the plane is given by the equation $x^3 - y^3 = 61$. What is the slope of the tangent line to the curve at the point (5,4)?

7. Find the area of the region in the first quadrant bounded by the curve $y = 9x - x^2$.

8. The region in the first quadrant bounded by the curves $y = 9x$ and $y = x^3$ is rotated about the y-axis. Find the volume of the resulting solid.

9. What is the center of mass of the triangle bounded by the coordinate axes and the line $2x + y = 1$?

10. Find $\displaystyle\int_0^{\pi/2} \sqrt{\sin x}\cos x dx$