Calculus I
Final Exam, Fall 2002

1. Find the derivatives of the following functions:
a) $f(x) = (x^2-3)^2(2x+4)$

b) $g(x) = \tan x \sin x$

2. Integrate:
a) $\int (6x^2+1)^5 x\,dx$

b) $\int \frac{dy}{y^{5/2}}$

3. Find the point in the first quadrant at which the tangent line to the curve $2y^2+6x-y=3$ has slope equal to 3.

4. Consider the region in the first quadrant bounded by the curve $y = 12 - \frac{3}{4}x^2$. What are the dimensions of the largest rectangle with sides parallel to the coordinate axes which can be inscribed inside this region?

5. Sketch the graph of the function $y = 3x^4 - 4x^3 - 12x^2 + 2$. Find the x values of all local minima, maxima and points of inflection.

6. Solve the initial value problem: $\frac{dy}{dx} = y^2 x \qquad y(0) = 2$

7. Find the area between the curve

$$y = \frac{x+1}{x^3}$$

and the x-axis, as x ranges from 1 to 4.

8. Find the volume of the solid obtained by rotating about the y-axis the region bounded by the curves $y = 4x$, $y = 5 - x^2$, $y = 0$.

9. Consider a curve given parametrically by $x = 4\cos^2 t$, $y = 3\sin^2 t$. Find the length of the piece of this curve running from $t = 0$ to $t = \pi/2$.

10. The square with vertices at (0,0),(1,0),(1,1),(0,1) is filled with a material whose density at the point (x,y) is $\delta(x,y) = x(1-x)$ g/cm^2. What is the mass of this object? What is its moment about the y-axis?