Calculus I
Exam 1, Spring 2003, Answers

1. Find the equation of the line which goes through the point (3,-2) and is parallel to the line given by the equation $2x-3y=1$.

Answer. Writing the equation of the given line as $y=(2/3)x-(1/3)$, we see it has slope $m=2/3$. Thus, the line we seek goes through (3,-2) and has slope 2/3, so has the equation

$$\frac{y+2}{x-3}=\frac{2}{3} \quad \text{or} \quad y=\frac{2}{3}x-4\ .$$

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2. Find the derivatives of the following functions:

a) $f(x)=3x^4-8x^2+x$

Answer. $f'(x)=3(4x^3)-8(2x)+1$, or $f'(x)=12x^3-16x+1$.

b) $g(x)=(x+1)(\frac{1}{x}+1)$

Answer. First write the function in exponential notation: $g(x)=(x+1)(x^{-1}+1)$ and now use the product rule:

$$g'(x)=(1)(x^{-1}+1)+(x+1)(-x^{-2})=1-x^{-2}\ .$$

c) $h(x)=\dfrac{x^2+1}{x+1}$

Answer. Use the quotient rule:

$$h'(x)=\frac{(x+1)(2x)-(x^2+1)}{(x+1)^2}=\frac{x^2+2x-1}{(x+1)^2}\ .$$

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3. Find the derivatives of the following functions:

a) $f(x)=(\tan(2x)+1)^2$

Answer. $f'(x)=4(\tan(2x)+1)(\sec^2(2x))$.

b) $g(x)=(2x^2+1)^{-1}$

Answer. $g'(x)=-(2x^2+1)^{-2}(4x)=\dfrac{-4x}{(2x^2+1)^2}$.

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4. Find the equation of the line tangent to the curve $y=(x^2+1)^2$ at (2,25).

Answer. Take differentials: $dy=2(x^2+1)(2x)dx$. Now, evaluate at $x=2$ and replace $dy,\ dx$ by $y-25,\ x-2$:

$$y-25=2(5)(4)(x-2) \quad \text{leading to} \quad y=40x-55\ .$$

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5. An object moves in a straight line so that its position at time t is given by $x(t) = t\cos t$. What is the velocity of the object when $t = 3\pi/4$?

Answer. Differentiating the velocity is $v = dx/dt = \cos t + t\sin t$. Now evaluate at $t = 3\pi/4$. The corresponding point is in the second quadrant, so

$$\cos(3\pi/4) = -\sqrt{2}/2,\ \sin(3\pi/4) = \sqrt{2}/2\ ,$$

so

$$v = -\frac{\sqrt{2}}{2} + \frac{3\pi}{4}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(\frac{3\pi}{4} - 1)\ .$$