Calculus I, Mathematics 1210-90

Examination 1, September 25,27, 2003, Answers

You may use graphing calculators. Each problem is worth 20 points. You MUST show your work. Just the correct answer is not sufficient for any points.

1. Find the derivatives of the following functions:

a) $f(x) = 3x^5 + 2x^2 - x + 1$

Solution. $f'(x) = 15x^4 + 4x - 1$.

b) $g(x) = (2x-1)^4(x-4)^5$

Solution. $g'(x) = 4(2x-1)^3(2)(x-4)^5 + (2x-1)^4(5)(x-4)^4 =$
$(2x-1)^3(x-4)^4(8(x-4)+5(2x-1)) = (2x-1)^3(x-4)^4(18x-37)$.

2. Find the derivatives of the following functions:

a) $f(x) = x^3 - x^{-3}$

Solution. $f'(x) = 3x^2 - (-3)x^{-4} = 3x^2 + 3x^{-4}$.

b) $g(x) = \dfrac{\tan x}{\sin x + 1}$

Solution. $g'(x) = \dfrac{(\sin x + 1)\sec^2 x - \tan x \cos x}{(\sin x + 1)^2}$.

3. Find the equation of the tangent line to the curve $y = x^3 - 3x + \sqrt{x}$ at the point (1,-1).

Solution. Write the function as $y = x^3 - 3x + x^{1/2}$.Then

$$\frac{dy}{dx} = 3x^2 - 3 + \frac{1}{2}x^{-1/2} .$$

At $x = 1$, $dy/dx = 3 - 3 + 1/2 = 1/2$. Thus the equation of the tangent line is

$$\frac{y+1}{x-1} = \frac{1}{2} \quad \text{or} \quad x - 2y = 3 .$$

4. Let $y = x^3 - 48x + 1$. Find the x coordinate of the points at which the graph has a horizontal tangent line.

Solution. The slope of the tangent line is the derivative $dy/dx = 3x^2 - 48 = 3(x^2 - 16)$. The tangent line is horizontal when its slope is 0, so we solve for $dy/dx = 0$, getting $x = \pm 4$.

5. A ball rolls down an inclined plane so that the distance travelled from time 0 to time t seconds is given by $s(t) = 48 - 6t - 12t^2$. What is its velocity at time $t = 3$ seconds?

Solution. The velocity is the derivative: $v = ds/dt = -6 - 24t$. At $t = 3$, we get $v = -6 - 24(3) = -78$.