

STATE MATH CONTEST PREPARATION

January 17, 2006

To get your brain cells jiggling after the long break, we are going to start by preparing for the upcoming state math contest. Below is a collection of problems from actual exams. I have grouped them thematically, and I've also added a couple of my own problems to each section. Many of the state math contest problems from previous years rely on a relatively small set of standard tricks. With a little practice, you'll be able to master those tricks.

**Recursion.**

1. Suppose  $f(n + 1) - f(n) = f(n) + 1$  and  $f(1) = 1$ . Find  $f(7)$ .
  - (a) 100
  - (b) 201
  - (c) 124
  - (d) 125
  - (e) 127

3. Let  $y_1 = f(x) = \frac{x+1}{x-1}$  and  $y_{n+1} = f(y_n)$  for  $n = 1, 2, 3, \dots$ . Then the value of  $y_{100}$  is

(a)  $x$

(b)  $\frac{x+1}{x-1}$

(c)  $\frac{x+100}{x-100}$

(d)  $\frac{100x+1}{100x-1}$

(e) None of the above.

4. Suppose  $f(n+1) - f(n) = n$  and  $f(1) = 1$ . Find  $f(10)$ .

5. Find a function  $f$  so that  $f(n+1) - f(n) = n$  and  $f(1) = 1$ .

6. Given a function  $f(x)$  satisfying

$$f(x) + 2f(1/(1-x)) = x,$$

find  $f(2)$ .

- (a)  $\frac{1}{2}$
- (b) 2
- (c) 3
- (d)  $\frac{6}{7}$
- (e) None of the above.

7. The value of

$$\sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{\dots}}}}$$

is

- (a) 3
- (b)  $\frac{1+\sqrt{13}}{2}$
- (c)  $\frac{1-\sqrt{13}}{2}$
- (d)  $\frac{3}{2}$
- (e) None of the above.

8. The value of

$$\sqrt[3]{6 + \sqrt[3]{6 + \sqrt[3]{6 + \sqrt[3]{\dots}}}}$$

is

- (a)  $\sqrt[3]{6}$
- (b)  $\frac{\sqrt[3]{6}+1}{2}$
- (c)  $\frac{1+\sqrt{37}}{2}$
- (d) 2
- (e) None of the above.

9. Evaluate

$$\frac{1}{3 + \frac{1}{3 + \frac{1}{\dots}}}$$

- (a) Same as the answer to 7.
- (b)  $\frac{1}{3}$
- (c)  $\frac{3+\sqrt{13}}{2}$
- (d)  $\frac{3-\sqrt{13}}{2}$
- (e) None of the above.

10. Evaluate

$$\frac{1}{2 + \sqrt{\frac{1}{2 + \sqrt{\frac{1}{2 + \dots}}}}}$$

- (a) 1
- (b) Same as answer to 9.
- (c)  $\sqrt[2]{\frac{1}{2}}$
- (d)  $\frac{2}{3}$
- (e) None of the above.