

INTRODUCTORY PROBLEMS

1. Prove the *rule of divisibility by 3*: an integer is divisible by 3 if and only if the sum of its digits is divisible by 3.
2. (a) Find the real solutions of the following polynomial equation:

$$(x - 3)^2 + (y - 2)^4 + (z + 5)^2 = 0$$

- (b) Does the following polynomial have any integral root?

$$x^4 - 3x^3 - 6x + 1$$

3. How many of the following are equal to $x^x + x^x$ for all $x \geq 0$?
 - (a) $2x^x$;
 - (b) x^{2x} ;
 - (c) $(2x)^x$;
 - (d) $(2x)^{2x}$;

4. How many prime numbers are factors of N if

$$\log_2(\log_3(\log_5(\log_7 N))) = 11?$$

5. Two non-zero real numbers, a and b , satisfy $ab = a - b$. Find the value of

$$\frac{a}{b} + \frac{b}{a} - ab.$$