MATH CIRCLE CONTEST
May 2, 2007

1.

Fred, the friendly gasoline distributor, has a tanker truck that has three tanks, two of whch are removable, the third of which is not. Each tank has a capacity of 1000 gallons, and Fred has a pump on his truck that allows him to transfer gas between any of the tanks. Unfortunately for Fred, his truck doesn't get very good gas mileage: he burns fuel at the steady rate of one gallon per mile traveled.

There is a full 10,000 gallon tank of gasoline in Adamsburg and an empty 10,000 gallon tank in Bakersfield. The distance between the two cities is exactly 3,000 miles.. Fred starts in Adamsburg at the 10,000 gallon tank with the three tanks of his truck bone dry. What is the maximum amount of fuel he can transport to Bakersfield?

Consider a six-by-six grid numbered as follows:

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

A coloring of the squares of the above by black and white squares is called *admissible* if there are exactly three black squares in each row and column. For example the standard alternating black-white ("checkerboard") coloring is an admissible coloring. Prove that the numbers in the white squares of *any* admissible coloring sum to 333.

3. Three's a charm?

The Borromean rings are a configuration of three rubber bands with the following properties:

(a) The bands are connected in the sense that one cannot remove a band from the configuration (without breaking one of the bands).

(b) If one breaks any single band, the entire configuration can be disassembled into two bands which are not connected to each other, together with the one broken band.

Prove or disprove: there exists a configuration of four rubber bands with the following properties:

(A) The bands are connected in the sense that one cannot remove a band from the configuration (without breaking one of the bands).

(B) If one breaks any single band, the entire configuration can be disassembled into three bands, none of which are connected to each other, and the one broken band.

4. A LONG DIVISION

Find the remainder when the polynomial

$$(x^2 + 1)^{100}$$

is divided by $x^4 + 1$.

5. Pen Pals

Each pair of 17 people exchange letters on one of three topics. Prove that there are at least 3 people who write to each other on the same topic.