Quick Review of the Conic Sections

a) Parabola
   \[ y = x^2 \quad x = y^2 \]

b) Ellipse
   \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

c) Hyperbola
   \[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]  \[ \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \]
Surfaces in Three-Space

The graph of a 3-variable equation which can be written in the form $F(x,y,z) = 0$ or sometimes $z = f(x,y)$ (if you can solve for $z$) is a surface in 3D. One technique for graphing them is to graph cross-sections (intersections of the surface with well-chosen planes) and/or traces (intersections of the surface with the coordinate planes).

We already know of two surfaces:

a) plane $Ax + By + Cz = D$

b) sphere $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$

EX 1 Sketch a graph of $z = x^2 + y^2$ and $x = y^2 + z^2$. 
A **cylinder** is the set of all points on lines parallel to \( \ell \) that intersect \( C \) where \( C \) is a plane curve and \( \ell \) is a line intersecting \( C \), but not in the plane of \( C \).

A **Quadric Surface** is a 3D surface whose equation is of the second degree. The general equation is

\[
Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0,
\]

given that \( A^2 + B^2 + C^2 \neq 0 \).

With rotation and translation, these possibilities can be reduced to two distinct types.

1) \( Ax^2 + By^2 + Cz^2 + J = 0 \)

2) \( Ax^3 + By^2 + Iz = 0 \)
Basic Quadric Surfaces

ELLIPSOID

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

HYPERBOLOID OF ONE SHEET

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \]

HYPERBOLOID OF TWO SHEETS

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \]

ELLiptic Paraboloid

\[ z = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]

HYPERBOLIC PARABOLOID

\[ z = \frac{y^2}{b^2} - \frac{x^2}{a^2} \]

ELLiptic Cone

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \]
EX 2 Name and sketch these graphs

a) $9x^2 + y^2 - z^2 = -4$

b) $9x^2 + y^2 - z^2 = 4$

c) $x^2 + 4y^2 - z = 0$

d) $x^2 + y^2 = 1$

e) $x^2 - y^2 = 25$

f) $z = y^3$