A 3-D curve can be given parametrically by \( x = f(t) \), \( y = g(t) \) and \( z = h(t) \) where \( t \) is on some interval \( I \) and \( f \), \( g \), and \( h \) are all continuous on \( I \).

We could specify the curve by the position vector
\[
\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}.
\]

Given a point \( P_0 \) determined by the vector, \( \mathbf{r}_0 \) and a vector \( \mathbf{v} = ai + bj + ck \), the equation
\[
\mathbf{r} = \mathbf{r}_0 + \mathbf{v} t
\]
determines a line passing through \( P_0 \) at \( t = 0 \) and heading in the direction determined by \( \mathbf{v} \).

(A special case is when you are given two points on the line, \( P_0 \) and \( P_1 \), in which case \( \mathbf{v} = \overrightarrow{P_0P_1} \).)

\[
\mathbf{r} = \langle x, y, z \rangle, \quad \mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle \Rightarrow \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + \langle a, b, c \rangle t
\]

\[
x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct
\]

These become the parametric equations of a line in 3D where \( a, b, c \) are called direction numbers for the line (as are any multiples of \( a, b, c \)).
EX 1 Find parametric equations of a line through 
$(2,-1,-5)$ and $(7,-2,3)$.

Symmetric Equations for a line

$x = x_0 + at, y = y_0 + bt, z = z_0 + ct \quad a \neq 0$

$t = \frac{x - x_0}{a} \quad t = \frac{y - y_0}{b} \quad t = \frac{z - z_0}{c}$

This is the line of intersection between the two planes given by

$\frac{x - x_0}{a} = \frac{y - y_0}{b} \quad \text{and} \quad \frac{y - y_0}{b} = \frac{z - z_0}{c}$.
EX 2 Write the symmetric equations for the line through (-2,2,-2) and parallel to 〈7,-6,3〉.

EX 3 Find the symmetric equations of the line through (-5,7,-2) and perpendicular to both 〈3,1,-3〉 and 〈5,4,-1〉.

EX 4 Find the symmetric equations of the line of intersection between the planes x + y - z = 2 and 3x - 2y + z = 3.
Tangent Line to a Curve

If \( \vec{r} = \vec{r}(t) = f(t) \hat{i} + g(t) \hat{j} + h(t) \hat{k} \) is a position vector along a curve in 3D,

\[
\vec{r}'(t) = \lim_{h \to 0} \frac{\vec{r}(t + h) - \vec{r}(t)}{h} \Rightarrow \vec{r}'(t) = f'(t) \hat{i} + g'(t) \hat{j} + h'(t) \hat{k}
\]

is a vector in the direction of the tangent line to the 3D curve. (This holds in 2D as well.)

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EX 5 Find the parametric equations of the tangent line to the curve

\[ x = 2t^2, \ y = 4t, \ z = t^3 \text{ at } t = 1. \]