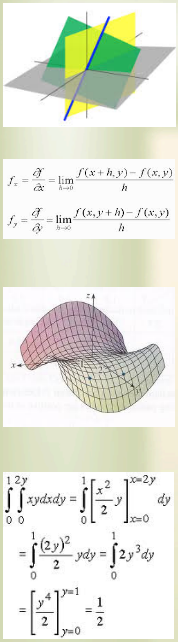
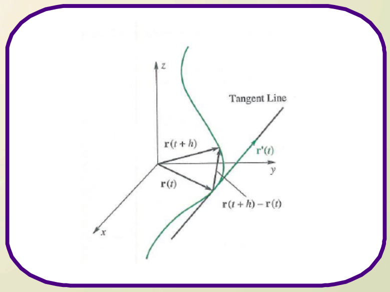


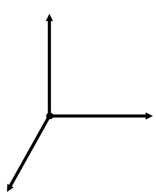
Lines and Tangent Lines in 3-Space



A 3-D curve can be given parametrically by $x = f(t)$, $y = g(t)$ and $z = h(t)$ where t is on some interval I and f, g, and h are all continuous on I.

We could specify the curve by the position vector

$\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$.



Given a point P_0, determined by the vector, $\vec{r}_0$ and a vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$, the equation $\vec{r} = \vec{r}_0 + \vec{v}t$ determines a line passing through P_0 at $t = 0$ and heading in the direction determined by $\vec{v}$.

(A special case is when you are given two points on the line, P_0 and P_1, in which case $\vec{v} = \overrightarrow{P_0P_1}$.)

$$\vec{r} = \langle x, y, z \rangle, \quad \vec{r}_0 = \langle x_0, y_0, z_0 \rangle \Rightarrow \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + \langle a, b, c \rangle t$$

$$x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$$

These become the parametric equations of a line in 3D where a,b,c are called direction numbers for the line (as are any multiples of a,b,c).

EX 1 Find parametric equations of a line through

(2,-1,-5) and *(7,-2,3)*.

<u>Symmetric Equations for a line</u>

$$x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct \qquad \begin{matrix} a \neq 0 \\ b \neq 0 \\ c \neq 0 \end{matrix}$$

$$\Rightarrow t = \frac{x - x_0}{a} \qquad t = \frac{y - y_0}{b} \qquad t = \frac{z - z_0}{c}$$

This is the line of intersection between the two planes given by

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} \quad \text{and} \quad \frac{y - y_0}{b} = \frac{z - z_0}{c} \ .$$

EX 2 Write the symmetric equations for the line through (-2,2,-2) and parallel to 〈7,-6,3〉.

EX 3 Find the symmetric equations of the line through (-5,7,-2) and perpendicular to both 〈3,1,-3〉 and 〈5,4,-1〉.

EX 4 Find the symmetric equations of the line of intersection between the planes $x + y - z = 2$ and $3x - 2y + z = 3$.

Tangent Line to a Curve

If $\vec{r} = \vec{r}(t) = f(t)\,\hat{i} + g(t)\,\hat{j} + h(t)\hat{k}$ is a position vector along a curve in 3D,

then $\vec{r}\,'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} \Rightarrow \vec{r}\,'(t) = f\,'(t)\hat{i} + g\,'(t)\hat{j} + h\,'(t)\hat{k}$

is a vector in the direction of the tangent line to the 3D curve. (This holds in 2D as well.)

EX 5 Find the parametric equations of the tangent line to the curve

$x = 2t^2,\ y = 4t,\ z = t^3$ at $t = 1$.