VECTORS (Geometric Approach)

Scalar

Vector

Magnitude

Direction

\[ \vec{u} = \vec{v} \] if they have the same magnitude and direction.

zero vector \( \vec{0} \) and \( \vec{0} + \vec{u} = \vec{u} + \vec{0} = \vec{u} \)

\( -\vec{u} \)

scalar multiple of \( \vec{u} = c\vec{u} \), where \( c \) is a real number,

means we have a vector in the direction of \( \vec{u} \) but scaled in length.
Adding vectors \( \Rightarrow \vec{u} + \vec{v} \)

EX 1
Express \( w \) in terms of \( u \) and \( v \).

EX 2
Draw \( w \) where \( w = v_1 + v_2 + v_3 \)
EX 3

Mark pushes on a post in the direction S 30° E with a force of 60 lbs. Dan pushes on the same post in the direction S 60° W with a force of 80 lbs. What are the magnitude and direction of the resulting force?

EX 4

A ship is sailing due south at 20 mph. A man walks west across the deck at 3 mph. What are the magnitude and direction of his velocity relative to the surface of the water?
Vectors (Algebraic Approach)

If we place our vector on a Cartesian Coordinate system
with its tail at the origin, then its head will end at some
point \((u_1, u_2, u_3)\). We say that \(\mathbf{u} = \langle u_1, u_2, u_3 \rangle\)

\(u_1, u_2\) and \(u_3\) are called components of \(\mathbf{u}\).

\[ u = v \text{ iff } u_1 = v_1, \ u_2 = v_2, \text{ and } u_3 = v_3 \]

\[ \mathbf{u} + v = \langle u_1, u_2, u_3 \rangle + \langle v_1, v_2, v_3 \rangle = \langle u_1+v_1, u_2+v_2, u_3+v_3 \rangle \]

\[ -\mathbf{u} = \langle -u_1, -u_2, -u_3 \rangle \quad c\mathbf{u} = \langle cu_1, cu_2, cu_3 \rangle \]

\[ 0 = 0\mathbf{u} = \langle 0, 0, 0 \rangle \]

Theorem A

For all vectors \(\mathbf{u}, \mathbf{v}, \mathbf{w}\) and the real numbers \(a\) and \(b\)

\[ \mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u} \]
\[ (\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}) \]
\[ \mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} \]
\[ \mathbf{u} + -\mathbf{u} = \mathbf{0} \]
\[ a(b\mathbf{u}) = (ab)\mathbf{u} \]
\[ a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v} \]
\[ (a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u} \]
\[ 1\mathbf{u} = \mathbf{u} \]

\[ ||\mathbf{u}|| = \sqrt{u_1^2 + u_2^2 + u_3^2} \]

\[ ||c\mathbf{u}|| = |c| ||\mathbf{u}|| \]
EX 5

Let \( \mathbf{u} = \langle -1, 5, 2 \rangle \), find \( ||\mathbf{u}|| \) and \( ||-3\mathbf{u}|| \).

Also, find a vector, \( \hat{\mathbf{u}} \) with the same direction as \( \mathbf{u} \) but with magnitude = 1. (This is called a unit vector)