Triple Integrals (Cylindrical and Spherical Coordinates)

\[
\iiint_V f(x, y, z) \, dV = \int_{\theta_0}^{\theta_1} \int_{\phi_0}^{\phi_1} \int_{r_0}^{r_1} f(r \cos \theta, r \sin \theta, z) r \, dz \, dr \, d\theta
\]

Note: Remember that in polar coordinates \( dA = r \, dr \, d\theta \)

**EX 1** Find the volume of the solid bounded above by the sphere \( x^2 + y^2 + z^2 = 9 \), below by the plane \( z = 0 \) and laterally by the cylinder \( x^2 + y^2 = 4 \). (Use cylindrical coordinates.)
EX 2 Find $\iiint_S f(x,y,z)dV$ for $f(x,y,z) = z^2 \sqrt{x^2+y^2}$ and

$S = \{(x,y,z)| x^2 + y^2 \leq 4, -1 \leq z \leq 3\}.$

Spherical Coordinates

$$\iiint_S f(x,y,z)dV = \int_0^\pi \int_{\theta}^{\pi} \int_{\phi}^{\phi} f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi \ d\rho \ d\theta \ d\phi$$

$$= \iiint_S f(x,y,z) dV$$

EX 3 Find $\iiint_S f(x,y,z)dV$ for $f(x,y,z) = x^2 + y^2$ on $S = \{(x,y,z)| x^2 + y^2 + z^2 \leq 1\}.$
EX 4 Find the volume of the solid inside the sphere
\[ x^2 + y^2 + z^2 = 16, \] outside the cone, \[ z = \sqrt{x^2 + y^2}, \]
and above the xy-plane.