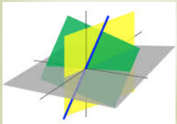
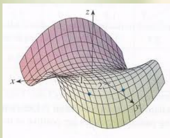
Triple Integrals in Cylindrical and Spherical Coordinates

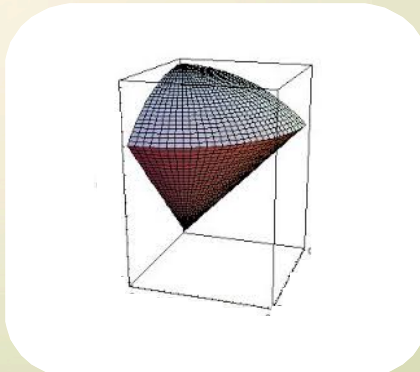


$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$



$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy = \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy = \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$



Triple Integrals (Cylindrical and Spherical Coordinates)

$$\iiint_S f(x,y,z)dV = \int_{\theta_1}^{\theta_{21}} \int_{r_1(\theta)}^{r_2(\theta)} \int_{g_1(r,\theta)}^{g_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$$

Note: Remember that in polar coordinates $dA = r\,dr\,d\theta$.

EX 1 Find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 9$, below by the plane $z = 0$ and laterally by the cylinder $x^2 + y^2 = 4$. (Use cylindrical coordinates.)

EX 2 Find $\iiint_S f(x,y,z)dV$ for $f(x,y,z) = z^2 \sqrt{x^2+y^2}$ and

$S = \{(x,y,z) | x^2 + y^2 \leq 4, -1 \leq z \leq 3\}$.

Spherical Coordinates

$$\iiint_S f(x,y,z)dV = \int_{\phi_1}^{\phi_2} \int_{g_1(\phi)}^{g_2(\phi)} \int_{\psi_1(\theta,\phi)}^{\psi_2(\theta,\phi)} f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi)\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \iiint_S f \; p^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

EX 3 Find $\iiint_S f(x,y,z)dV$ for $f(x,y,z) = x^2 + y^2$ on $S = \{(x,y,z) | x^2 + y^2 + z^2 \leq 1\}$.

EX 4 Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$, outside the cone, $z = \sqrt{x^2 + y^2}$, *and above the* xy-plane.