

$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{0}{0}$
 or
 $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then

$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

$f(x) = f(x) + f(x)(x-a) + \frac{f''(x)}{2!}(x-a)^2 + \dots$
 $\frac{f'(x)}{g'(x)} = \frac{f'(x) + f''(x)(x-a) + \frac{f'''(x)}{3!}(x-a)^2 + \dots}{g'(x) + g''(x)(x-a) + \frac{g'''(x)}{3!}(x-a)^2 + \dots}$



$u(x) = \frac{1}{x} = u(x) + u'(x)(x-a) + \frac{u''(x)}{2!}(x-a)^2 + \dots$



$\int u dv = uv - \int v du$

Example: $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

Integration by Parts

$$\int u dv = uv - \int v du$$

Use the product rule for differentiation

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Integrate both sides $\int \frac{d}{dx}(uv) = \int \left(u \frac{dv}{dx} + v \frac{du}{dx} \right)$

$$uv = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$$

Simplify

Rearrange

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int u dv = uv - \int v du$$

Use the product rule for differentiation

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Integrate both sides $\int \frac{d}{dx}(uv) = \int \left(u \frac{dv}{dx} + v \frac{du}{dx} \right)$

Simplify

$$uv = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$$

Rearrange

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

Integration by Parts

Look at the Product Rule for Differentiation.

$$D_x[u(x)v(x)] = u'(x)v(x) + v'(x)u(x)$$

EX 1 $\int x \sin(2x) dx$

EX 2 $\int \arctan(5x) dx$

EX 3 $\int \frac{\ln x}{\sqrt{x}} dx$

Repeated Integration by Parts

EX 4 $\int x^3 e^x dx$

EX 5 $\int e^x \cos x dx$