Taylor and Maclaurin Series

If we represent some function $f(x)$ as a power series in $(x-a)$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x-a)^n$$
**Uniqueness Theorem**

Suppose

for every $x$ in some interval around $a$.

Then .

**Taylor’s Formula with Remainder**

Let $f(x)$ be a function such that $f^{(n+1)}(x)$ exists for all $x$ on an open interval containing $a$.

Then, for every $x$ in the interval,

where $R_n(x)$ is the remainder (or error).

**Taylor’s Theorem**

Let $f$ be a function with all derivatives in $(a-r,a+r)$.

The Taylor Series represents $f(x)$ on $(a-r,a+r)$

if and only if .
EX 1 Find the Maclaurin series for $f(x) = \cos x$ and prove it represents $\cos x$ for all $x$.

EX 2 Find the Maclaurin series for $f(x) = \sin x$. 
EX 3  Write the Taylor series for $f(x) = \frac{1}{x}$ centered at $a=1$.

EX 4  Find the Taylor series for $f(x) = \sin x$ in $x - \pi/4$. 
EX 5 Use what we already know to write a Maclaurin series (5 terms)

for .