Positive Series: Integral Test

Bounded Sum Test
A series $\sum a_i$ of nonnegative terms converges if and only if its partial sums are bounded above.

EX 1 Does $\sum_{k=1}^{\infty} \frac{|\sin k|}{(k + 1)!}$ converge?
Integral Test

If \( f(x) \) is continuous, positive and nonincreasing on \([N, \infty)\)
and \( a_k = f(k) \) for all positive integers, \( k \), then
\[
\sum_{n=N}^{\infty} a_n \text{ converges if and only if } \int_{N}^{\infty} f(x) \, dx \text{ converges.}
\]

EX 2 Does \[
\sum_{k=1}^{\infty} \frac{5k^2}{1 + k^3}
\] converge or diverge?

\( p \)-series test

\[
\sum_{k=1}^{\infty} \frac{1}{k^p}
\] is called a \( p \)-series. It converges if \( p > 1 \) and diverges if \( p \leq 1 \).
EX 3 Does $\sum_{k=1}^{\infty} \frac{1}{k^3}$ converge or diverge?

EX 4 Estimate the error made by approximating the series by the sum of the first five terms.

$$E_n = \sum_{k=1}^{n} \frac{1}{k^{3/2}} \quad S_n = \sum_{k=1}^{n} \frac{1}{k^{3/2}}$$