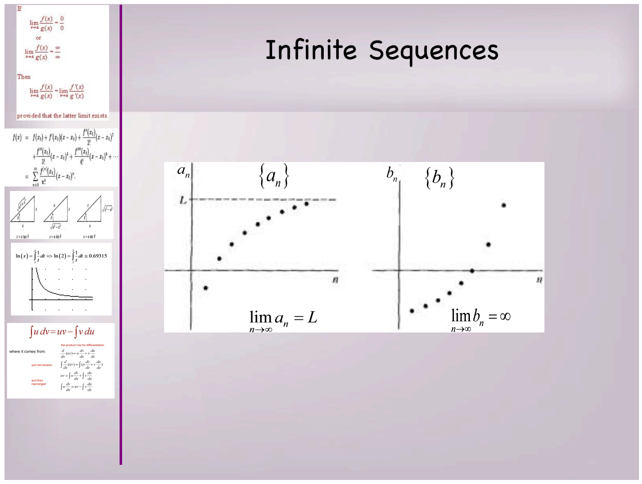


<u>Definition</u>

An <u>infinite sequence</u> is an ordered arrangement of real numbers.

$$a_1, a_2, a_3, a_4, ...$$

$$\{a_n\}_{n=1}^{\infty}$$

$$\{a_n\}$$

iteration (explicit) vs recursion (implicit)

$$a_n = 5n - 3$$

$$a_1 = 2$$

$$a_n = a_{n-1} + 5 \qquad n \geq 2$$

We can just write out the terms.

$$2, 7, 12, 17, 22, ...$$

Definition

Convergence

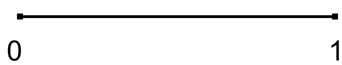
$\{a_n\}$ converges to L, written $\lim_{n \to \infty} a_n = L$

if for each positive ε there exists a corresponding positive N such that

$n \geq N \Rightarrow |a_n - L| < \varepsilon$.

If a sequence fails to converge to a finite L, then it diverges.

Example $a_n = \dfrac{n}{2n-1}$



EX 1 Does $\{a_n\}$ converge? $a_n = \dfrac{5n^2 - 3n + 1}{2n^2 + 7}$

If so, what is the limit?

Properties of limits of sequences

Assume $\lim\limits_{n\to\infty} a_n$ and $\lim\limits_{n\to\infty} b_n$ exist, then

1) $\lim\limits_{n\to\infty} k = k$

2) $\lim\limits_{n\to\infty}(a_n \cdot b_n) = \left(\lim\limits_{n\to\infty} a_n\right)\left(\lim\limits_{n\to\infty} b_n\right)$

3) $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{\lim\limits_{n\to\infty} a_n}{\lim\limits_{n\to\infty} b_n}, \quad b_n \neq 0$

4) $\lim\limits_{n\to\infty} k a_n = k \lim\limits_{n\to\infty} a_n$

5) $\lim\limits_{n\to\infty}(a_n \pm b_n) = \lim\limits_{n\to\infty} a_n \pm \lim\limits_{n\to\infty} b_n$

If $\lim\limits_{x\to\infty} f(x) = L$, then $\lim\limits_{n\to\infty} f(n) = L$.

x is a continuous variable

n is a discrete variable

We can use l'Hopital's Rule.

EX 2 Determine if $\{a_n\}$ converges and if so, find $\lim\limits_{n\to\infty} a_n$.

a) $a_n = \dfrac{\ln(1/n)}{\sqrt{2n}}$

b) $a_n = \dfrac{n^{100}}{e^n}$

Squeeze Theorem

If $\{a_n\}$ and $\{c_n\}$ both converge to L and

$a_n \le b_n \le c_n$ for $n \ge K$ (some fixed integer),

then $\{b_n\}$ also converges to L.

EX 3 Determine if $\{a_n\}$ converges and if so, find $\lim_{n\to\infty} a_n$.

$\{a_n\} = e^{-n}\sin n$

Theorem

If $\lim_{n\to\infty}|a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

EX 4 Show that if r is in this interval $(-1,1)$, then

$\lim_{n\to\infty} r^n = 0$.

Monotonic Sequence Theorem

If U is an upper bound for a nondecreasing sequence $\{a_n\}$,then the sequence converges to a limit A such that $A \leq U$.

Also, if L is a lower bound for a nonincreasing sequence $\{b_n\}$, then the sequence converges to a limit B such that $B \geq L$.

EX 5 Write the first four terms for this sequence. Show that it converges.

$$\{a_n\} = \frac{n}{n+1}\left(2 - \frac{1}{n^2}\right)$$