Calculus: The Slope of A Curve

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ \frac{d}{dx} \int_a^x f(t) \, dt = f(x) \]

\[ \lim_{n \to \infty} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) \, dx \]

\[ \int_a^b f(x) \, dx = F(b) - F(a) \]
How do we find the slope of a curve?

Try to find the slope of this curve at the point (1, 1).

First point (1, 1)

Second point: Slope at that point:

<table>
<thead>
<tr>
<th>First point</th>
<th>Slope of line through (1, 1) and this pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 9)</td>
<td>$m = \frac{9-1}{3-1} = 4$</td>
</tr>
<tr>
<td>(2, 4)</td>
<td>$m = \frac{4-1}{2-1} = 3$</td>
</tr>
<tr>
<td>(1.1, 1.21)</td>
<td>$m = \frac{1.21-1}{1.1-1} = 0.21$</td>
</tr>
<tr>
<td>(1.01, 1.0201)</td>
<td>$m = \frac{1.0201-1}{1.01-1} = 0.01$</td>
</tr>
</tbody>
</table>

Slope to the left of the origin?

Always negative (but changing)

Slope to the right of the origin?

Always positive (changing)

$h = \text{horizontal distance from } x = 1$

Slope between (1, 1) and (1 + h, (1 + h)^2): $m = \frac{(1 + h)^2 - 1}{1 + h - 1} = \frac{1 + 2h + h^2 - 1}{h}$

$= \frac{2h + h^2}{h} = \frac{h(2h)}{h}$

$= 2h$

As $h$ gets very small, slope becomes 2
EX 1
Find the slope of the curve \( y = x^2 - 5x \) at (2, -6)

*hint: Calculate the slope between (2, -6) and (2+h, f(2+h))*

\[
m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{(2+h)^2 - 5(2+h) + 6}{h}
\]

\[
= \lim_{h \to 0} \frac{4 + 4h + h^2 - 10 - 5h + 6}{h}
\]

\[
= \lim_{h \to 0} \frac{h^2 - h}{h} = \lim_{h \to 0} h(h-1) = \lim_{h \to 0} (h-1)
\]
7B Slope of Curve

Definition: The slope of a function, $f$, at a point $x = (x, f(x))$ is given by

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x)$ is called the derivative of $f$ with respect to $x$.

Other names for $f'(x)$:
- slope
- instantaneous rate of change
- speed
- velocity

EX 2
Find the derivative of $f(x) = 4x - 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If I plug in $h = 0$, we get

$$= \lim_{h \to 0} \frac{4(x+h) - 1 - (4x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{4x + 4h - 1 - 4x + 1}{h}$$

$$= \lim_{h \to 0} \frac{4h}{h} = \lim_{h \to 0} 4 = 4$$

(Note: this is a line, $m \parallel \text{slope } 4$)
EX 3
Find the derivative of \( f(x) = x^2 + 4x - 1 \)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - 1 - (x^2 + 4x - 1)}{h}
\]

\[
= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - 1 - x^2 - 4x + 1}{h}
\]

\[
= \lim_{h \to 0} \frac{h(2x + h + 4)}{h} = \lim_{h \to 0} 2x + h + 4 = 2x + 4
\]

\[
f'(x) = 2x + 4
\]
What is the slope at point $P$?

Tangent line is horizontal

\[ m = 0 \]