Definition: (Limit as \( x \to \infty \))

is defined on \([c, \infty)\) for \(c \in \mathbb{R}\)

We say that if for every \(\varepsilon > 0\) there is a corresponding number, \(m\) such that
EX 1  Intuitively (looking at the graph) determine these limits.

\[
\lim_{x \to 0} f(x) =
\]

\[
\lim_{x \to \infty} f(x) =
\]

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EX 2  Show that if \( n \) is a positive integer, then

\[
\lim_{x \to \infty} \frac{1}{x^n} = 0
\]

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EX 3  \[
\lim_{x \to 0} \frac{2x + 3}{x^2 + 1} =
\]

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EX 4  \[
\lim_{x \to \infty} \frac{3x^4 - 2x^3 + 53}{x^3 + 7} =
\]

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#X 5  \[
\lim_{x \to \infty} \frac{2x^2 + 5x - 1}{x^2 + 3x} =
\]
Definition: (Infinite limit)
We say \( \lim_{x \to c} f(x) = \infty \) if for every positive number, \( m \)
there is a corresponding \( \delta > 0 \) such that \( 0 < x - c < \delta \Rightarrow f(x) > m \)

EX 6 Determine these limits looking at this graph of \( f(x) = \frac{1}{x-1} \).

\[
\begin{align*}
\lim_{x \to 0} f(x) &= \\
\lim_{x \to 1} f(x) &= \\
\lim_{x \to \infty} f(x) &= \\
\lim_{x \to -\infty} f(x) &= 
\end{align*}
\]

Ex 7 Find the horizontal and vertical asymptotes for this function, then write a few limit statements including \( \infty \).

\( f(x) = \frac{-2x}{x + 3} \)
Ex 8  a) Find the vertical and horizontal asymptotes for this function.

\[ f(x) = \frac{2x}{\sqrt{x^2} + 5} \]

b) Determine these limits:

\[ \lim_{x \to \infty} f(x) = \quad \lim_{x \to 5} f(x) = \]

\[ \lim_{x \to -\infty} f(x) = \quad \lim_{x \to -3} f(x) = \]

Determine these limits:

\[ \lim_{x \to 0^+} f(x) = \]

\[ \lim_{x \to 0^-} f(x) = \]

\[ \lim_{x \to 0^+} f(x) = \]

\[ \lim_{x \to 0^-} f(x) = \]