Moments, Center of Mass

Two Children of Equal Mass

One Highschooler and One Elementary Schooler

One Baby and One Cow

One Planet and One Star

\[ f(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

\[ \frac{d}{dx} \int_a^b f(t) \, dt = f(x) \]

\[ \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \approx \int_a^b f(x) \, dx \]

\[ \int_a^b f(x) \, dx = F(b) - F(a) \]
The moment of a particle with respect to a point is the product of mass \((m)\) of the particle with its directed distance \((x)\) from a point. This measures the tendency to produce a rotation about that point.

Total moment \((M)\) for a bunch of masses = \[ \sum x_i m_i \]

Where does the fulcrum need to be placed to balance? Let’s call it \(\bar{x}\).

EX 1

John and Mary, weighing 180 lbs and 110 lbs respectively, sit at opposite ends of a 12-ft teeter-totter with the fulcrum in the middle. Where should their 50-lb son sit in order for the board to balance?
For a continuous mass distribution along the line (like on a wire):

\[
\bar{x} = \frac{M}{m} = \frac{\int_a^b x \delta(x) \, dx}{\int_a^b \delta(x) \, dx}
\]

\((\delta(x) \text{ is density fn})\)

Since total mass is \(\int_a^b \delta(x) \, dx\)

and moment is \(\int_a^b x \delta(x) \, dx\)

EX 2

A straight wire 7 units long has density \(\delta(x) = 1 + x^4\) at a point \(x\) units from one end. Find the distance from this end to the center of mass.
Consider a discrete set of 2-d masses.

How do we find the center of mass (the geometric center) \((\bar{x}, \bar{y})\)?

\[
\begin{align*}
\bar{x} &= \frac{\sum x_i m_i}{m} \\
\bar{y} &= \frac{\sum y_i m_i}{m}
\end{align*}
\]

where \(m = \sum m_i\),

\[
M_x = \sum y_i m_i \\
M_y = \sum x_i m_i
\]

EX 3

The masses and coordinates of a system of particles are given by the following:

5, (-3,2); 6, (-2,-2); 2, (3,5); 7, (4,3); 1, (7,-1). Find the moments of this system with respect to the coordinate axes and find the center of mass.
Now, consider a continuous 2-d region (a lamina) that has constant (homogeneous) density everywhere. How do we find the center of mass \((\overline{x}, \overline{y})\)?

It's still true \(\overline{x} = \frac{M_y}{m}, \overline{y} = \frac{M_x}{m}\)

EX 4
Find the centroid of the region bounded by \(y = x^2\) and \(y = x+2\).
33 Moment Center of Mass

Two Children of Equal Mass

One Highschooler and One Elementary Schooler

One Baby and One Cow

One Planet and One Star

Photo source: Laboratory for Atmospheric and Space Physics, University of Colorado at Boulder