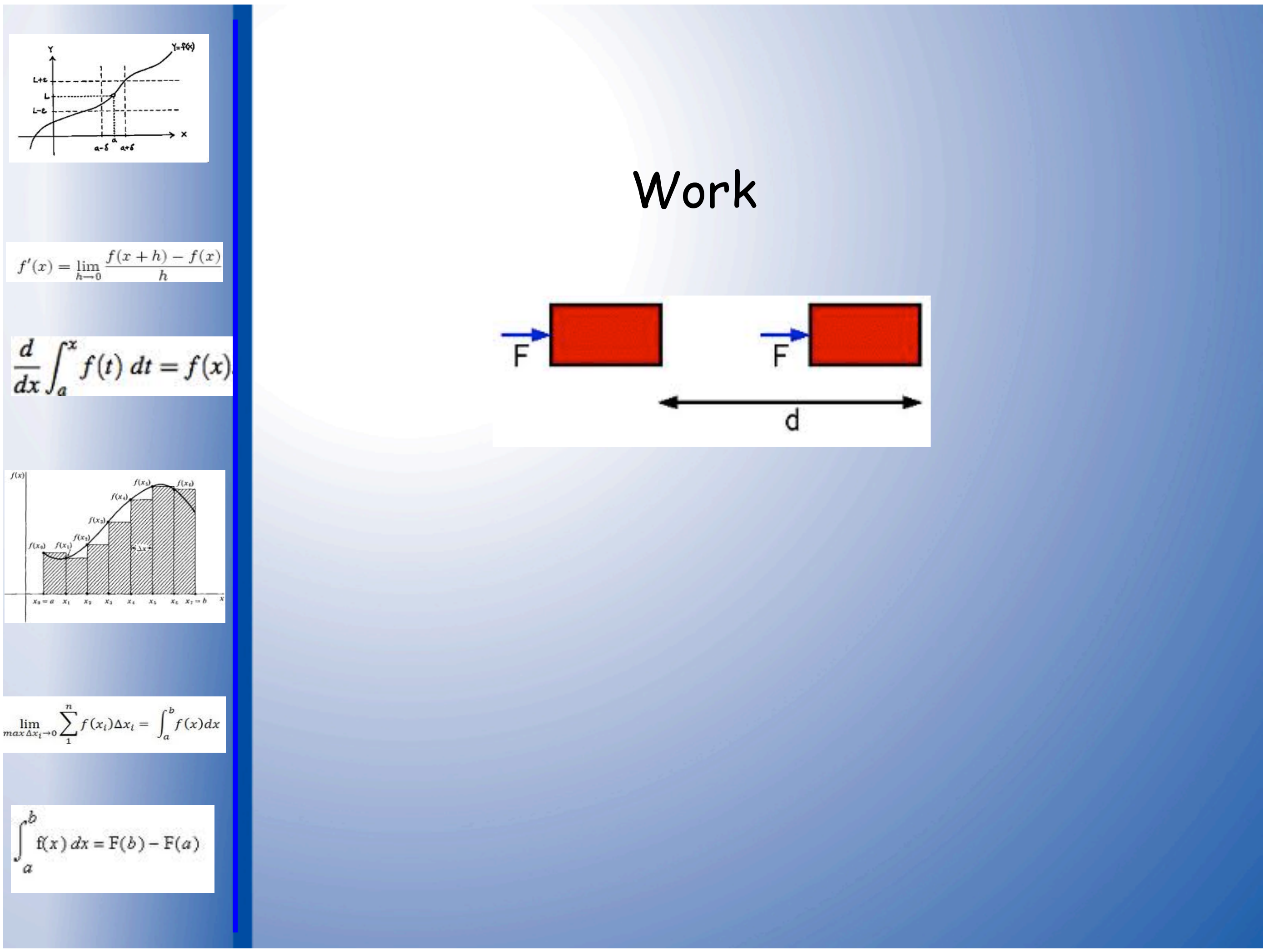

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$
Work
F
F
d

Work = Force · Distance

$W = Fd$

If force is measured in Newtons, distance is measured in meters, then work units are joules.
If force is measured in pounds, distance is measured in feet, then work units are foot-pounds.

Force is sometimes variable, in which case we need to approximate the work done in little 'chunks' and then add up all the 'chunks' of work.

Aha, another case for the definite integral!

$$W = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \underbrace{F(x_i)}_{\text{force at } x_i} \underbrace{\Delta x}_{\text{little bit of distance}}$$

$$\boxed{W = \int_a^b F(x)\,dx}$$

Springs

Hooke's law says $F(x) = kx$ where k is a constant for that spring, and $F(x)$ is the force necessary to keep the spring stretched (or compressed) x units beyond (or short of) its natural length.

 natural length

EX 1 A force of 6 lbs is required to keep a spring stretched 1/2 ft beyond its normal length.

a) Find the spring constant.

$F = kx$ when $F = 6$ lbs, $x = \frac{1}{2}$ ft

$6 = k\left(\frac{1}{2}\right) \Rightarrow \boxed{k = 12}$

b) Find the work done in stretching the spring 1/2 ft beyond its natural length.

$W = \int_a^b F(x)\,dx$, and we have $F(x) = 12x$

$x \in [0, \frac{1}{2}]$

$$W = \int_0^{1/2} 12x\,dx$$

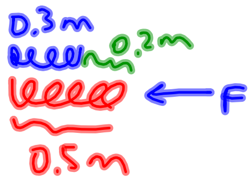
$$= \frac{12x^2}{2}\Big|_0^{1/2}$$

$$= 6\left(\left(\frac{1}{2}\right)^2 - 0^2\right)$$

$$= \frac{6}{4} = \boxed{\frac{3}{2}\text{ ft-lbs}}$$

EX 2 ① A force of 1.8 newtons is required to keep a spring of natural length of 0.5 m compressed to a length of 0.3 m. ② Find the work done in compressing the spring from its natural length to a length of 0.2 m.

x = distance from natural length to compression



① Find spring constant. $F = kx$

$$1.8 = k(0.2)$$

$$k = 9$$

$$\Rightarrow F = 9x$$

$$② \Rightarrow W = \int_a^b 9x\,dx = \int_0^{0.3} 9x\,dx$$

$$= \frac{9x^2}{2}\Big|_0^{0.3}$$

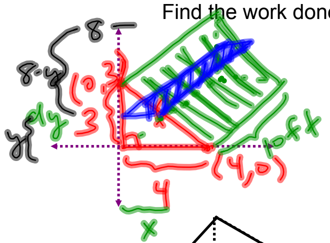
$$= \frac{9}{2}\left((0.3)^2 - 0^2\right)$$

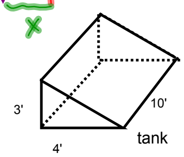
$$= \frac{9}{2}(0.09) = 4.5(0.09)$$

$$= \boxed{0.405 \text{ joules}}$$

29B Work

EX 3 A tank with the triangular cross-section has a length of 10 ft and is full of water. The water is to be pumped to a heigh of 5 ft above the top of the tank. Find the work done in emptying the tank.





eqn of line ℓ: $(0,3)$ $(4,0)$

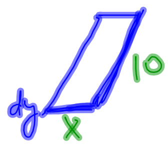
$$m = \frac{3-0}{0-4} = \frac{-3}{4} \qquad \boxed{y = \frac{-3}{4}x + 3}$$

or $y - 3 = \frac{-3}{4}x$ or $\boxed{x = \frac{-4}{3}y + 4}$

$W = Fd$ in this case $F = \text{Volume} \cdot \delta$ (δ = density of water; $\delta = 6.24\ lb/ft^3$)

$W = V\delta d$

$$V_{slab} = x\,dy\,(10) = \left(\frac{-4}{3}y + 4\right)(10)\,dy$$



$d = 8 - y$

$$W = \int_0^3 \underbrace{\delta\left(\frac{-4}{3}y+4\right)(10)\,dy}_{\text{force}}\ \underbrace{(8-y)}_{\text{distance}}$$

$$= 10\delta \int_0^3 \left(\frac{-32}{3}y + \frac{4}{3}y^2 + 32 - 4y\right)dy$$

$$= 10\delta \int_0^3 \left(\frac{4}{3}y^2 - \frac{44}{3}y + 32\right)dy$$

$$= 10\delta \left(\frac{4}{3}\left(\frac{y^3}{3}\right) - \frac{\cancel{44}^{22}}{3}\left(\frac{y^2}{\cancel{2}_1}\right) + 32y\right)\Big|_0^3$$

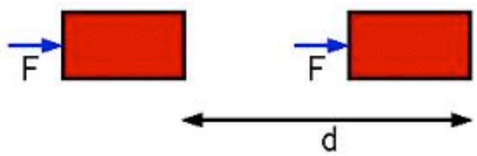
$$= 10\delta\left(\frac{4}{\cancel{9}}(\cancel{3^3}^{3}) - \frac{22}{\cancel{3}}(\cancel{3^2}) + 32(3)\right) - 0\Big)$$

$$= 10\delta(12 - 22(3) + 96)$$

$$= 10\delta(108 - 66)$$

$$= 10\delta(42) = 420\delta = 420(6.24)$$

$$= \boxed{26208 \text{ ft-lbs}}$$


F
F
d