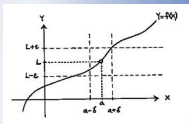
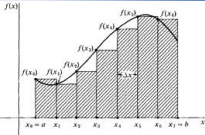


## 24 Introduction to Area



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

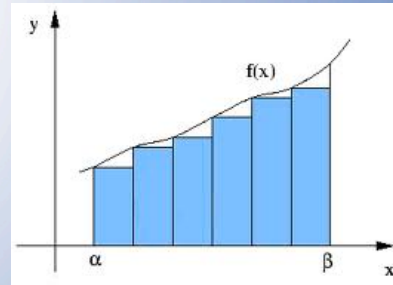
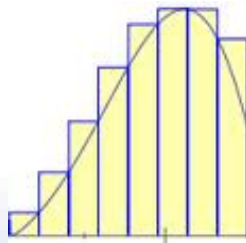
$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



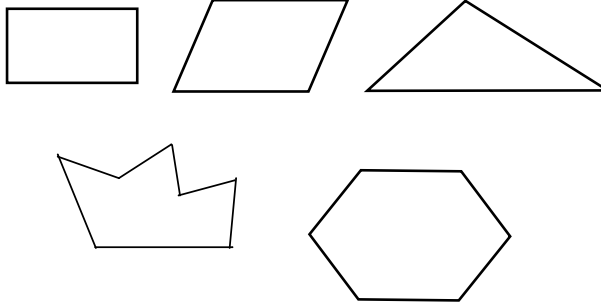
$$\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

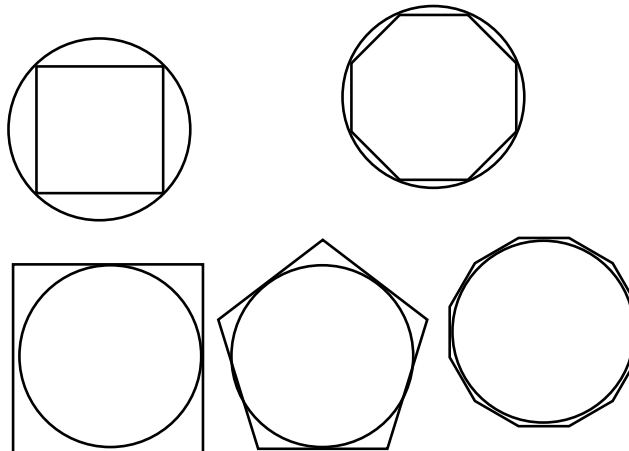
## Introduction to Area



Area of a Polygon:



Estimating the area of a circle:



## 24 Introduction to Area

Sums and Sigma Notation

$$1+2+3+4+\dots+100 =$$

$$2+4+6+8+\dots+1000 =$$

$$1+4+9+16+\dots+625 =$$

$\Sigma$  = Sigma, the capitol Greek letter called "sigma";  
It means summation.  $i$  = index

$$\sum_{j=1}^n \frac{1}{j} =$$

$$\sum_{i=1}^n c =$$

Linearity of  $\Sigma$

Let  $\{a_i\}$  and  $\{b_i\}$  denote two sequences and  $c$  is a real number.

$$(i) \sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$$

$$(ii) \sum_{i=1}^n a_i \pm b_i = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

### Special Sum Formulas

$$\sum_{i=1}^n i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

$$\sum_{i=1}^n i^4 = 1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30}$$

## 24 Introduction to Area

$$\text{EX 1} \quad \sum_{i=1}^{10} [(i-1)(4i+3)]$$

$$\text{EX 2} \quad \sum_{j=1}^n (2j-3)^2$$

$$\text{EX 5} \quad \text{Change the variable in the index to start at 1.} \quad \sum_{k=5}^{14} k2^{k-4}$$

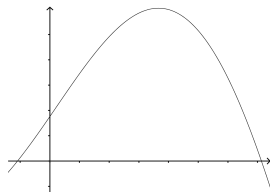
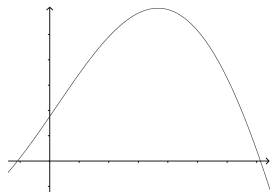
Collapsing Sum  $\sum_{i=1}^n (a_{i+1} - a_i) = a_{n+1} - a_1$

$$\text{EX 3} \quad \sum_{k=1}^{10} (2^k - 2^{k-1})$$

$$\text{EX 4} \quad \sum_{k=3}^{m+1} (a_k - a_{k-1})$$

## 24 Introduction to Area

We will estimate the area under a curve using inscribed or circumscribed rectangles.



EX 6 For  $f(x)=3x-1$ , divide the interval  $[1,3]$  into 4 equal subintervals. Calculate the area of the circumscribed rectangles.

