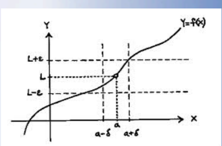
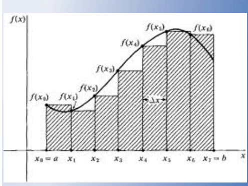



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$



$$\lim_{max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

# Differential Equations

EXAMPLE: $\frac{dy}{dx} = \frac{x^2}{y}$

SOLUTION:

$$y dy = x^2 dx$$

$$\int y dy = \int x^2 dx$$

$$\frac{1}{2} y^2 = \frac{1}{3} x^3 + C$$

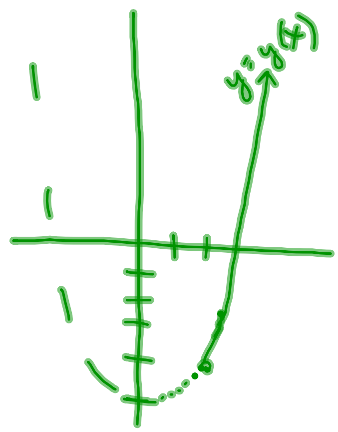
$$y^2 = \frac{2}{3} x^3 + C$$

$$y = \pm \sqrt{\frac{2}{3} x^3 + C}$$

**23B Differential Equations**

A **differential equation** is an equation that contains a derivative. We will need to integrate both sides, at some point, to 'undo' the derivative.

EX 1 Find the equation of the curve that goes through the point (2,-4) and whose slope at any point on the curve is 3x.




$$\frac{dy}{dx} = 3x$$

$$dy = 3x\,dx$$

$$\int dy = \int 3x\,dx$$

$$y = 3\left(\frac{x^2}{2}\right) + c$$

$$y = \frac{3}{2}x^2 + c$$ (general soln)

plug in pt to solve for c:
(2,-4)

$$-4 = \frac{3}{2}(2^2) + c$$

$$-4 = 6 + c \Rightarrow c = -10$$

$$\Rightarrow \boxed{y = \frac{3}{2}x^2 - 10}$$ (particular soln)

EX 2 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y = 4$ *when* $x = 1$ $(1, 4)$

$$\frac{dy}{dx} = \frac{\sqrt{x}}{\sqrt{y}}$$

$$\sqrt{y}\,dy = \sqrt{x}\,dx$$

$$\int y^{1/2}\,dy = \int x^{1/2}\,dx$$

$$\frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + C$$

note:

$$\frac{2}{3}y^{3/2} + C_1 = \frac{2}{3}x^{3/2} + C_2$$

$$\frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + \underbrace{(C_2 - C_1)}_{C}$$

plug in $(1,4)$ to solve for $C$:

$$\frac{2}{3}(4)^{3/2} = \frac{2}{3}(1)^{3/2} + C$$

$$\frac{2}{3}(8) = \frac{2}{3} + C$$

$$\frac{16}{3} - \frac{2}{3} = C \Leftrightarrow C = \frac{14}{3}$$

$$\Rightarrow \frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + \frac{14}{3}$$

$$\boxed{y^{3/2} = x^{3/2} + 7}$$

or $y = \left(x^{3/2} + 7\right)^{2/3}$

EX 3 $\frac{dy}{dx} = -y^2\left(x^2+2\right)^4 x$ *through* $(0,1)$

$$\frac{-1}{y^2}\,dy = (x^2+2)^4 x\,dx$$

$$\int -y^{-2}\,dy = \int (x^2+2)^4 x\,dx$$

$$-\left(\frac{y^{-1}}{-1}\right) = \frac{1}{10}(x^2+2)^5 + c$$

$$\frac{1}{y} = \frac{1}{10}(x^2+2)^5 + c$$

(general soln)

$$\int (\underbrace{x^2+2}_{u})^4 \underbrace{x\,dx}$$

$u = x^2+2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int u^4\left(\tfrac{1}{2}\right)du$$

$$= \frac{1}{2}\left(\frac{u^5}{5}\right) + c$$

$$= \frac{1}{10}(x^2+2)^5 + c$$

thru $(0,1)$:

$$\frac{1}{1} = \frac{1}{10}(0+2)^5 + c$$

$$1 = \frac{16}{5} + c$$

$$\frac{-11}{5} = c \Rightarrow \boxed{\frac{1}{y} = \frac{1}{10}(x^2+2)^5 - \frac{11}{5}}$$

$$\frac{1}{y} = \frac{(x^2+2)^5 - 22}{10}$$

$$y = \frac{10}{(x^2+2)^5 - 22}$$

EX 4 The acceleration of an object moving along a coordinate line is $a(t)=18(t-3)^{-3}$ in meters per second per second.

a) If the velocity at $t=0$ is $4$ meters per second, find the velocity 2 seconds later.

b) If the initial position is $-3$ m, find an equation for the position of the object at time, t.

$a(t)=v'(t)$ & $v(t)=s'(t)$

(a) $a(t)=18(t-3)^{-3}=\frac{dv}{dt}$

$18(t-3)^{-3}\,dt=dv$

$\int 18(t-3)^{-3}\,dt=\int dv$

$\int 18(t-3)^{-3}\,dt$

$u=t-3$

$\frac{du}{dt}=1 \Rightarrow du=dt$

$=\int 18(u)^{-3}\,du$

$=18\left(\frac{u^{-2}}{-2}\right)+c$

$=-9(t-3)^{-2}+c$

$-9(t-3)^{-2}+c=v$

need to solve for c: $v=4$, when $t=0$

$4=-9(-3)^{-2}+c$

$4=\frac{-9}{(-3)^2}+c$

$4=-1+c \Rightarrow c=5$

$$\boxed{v=\frac{-9}{(t-3)^2}+5}$$

want to know $v(2)$

$v(2)=\frac{-9}{(2-3)^2}+5=-9+5=\boxed{-4\ m/s}$

(b) $t=0$ sec, $s=-3$ m

$s'(t)=v(t) \Leftrightarrow \frac{-9}{(t-3)^2}+5=\frac{ds}{dt}$

$\int\left(-9(t-3)^{-2}+5\right)dt=\int ds$

$\frac{-9(t-3)^{-1}}{-1}+5t+c=s$

$9(0-3)^{-1}+5(0)+c=-3$

$9\left(\frac{-1}{3}\right)+c=-3$

$-3+c=-3 \Rightarrow c=0$

$$\Rightarrow \boxed{s(t)=9(t-3)^{-1}+5t} \text{ m}$$

EXAMPLE: $\frac{dy}{dx} = \frac{x^2}{y}$

SOLUTION:

$$y\,dy = x^2\,dx$$

$$\int y\,dy = \int x^2\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$$

$$y^2 = \frac{2}{3}x^3 + C$$

$$y = \pm\sqrt{\frac{2}{3}x^3 + C}$$