Solving Equations Numerically

Three numeric methods for solving an equation numerically:

① Bisection Method
② Newton's Method
③ Fixed-point Method
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① Bisector Method Algorithm
Let $f(x)$ be a continuous function and let $a_i$ and $b_i$ be numbers satisfying $a_i < b_i$ and $f(a_i) f(b_i) < 0$.
Let $E$ denote the desired bound for the error $|r-m|$.
Repeat steps 1 to 5 for $n=1,2,...$ until $h_n < E$

1. Calculate $m_n = \frac{a_n + b_n}{2}$
2. Calculate $f(m_n)$ and if $f(m_n) = 0$ then STOP.
3. Calculate $h_n = \frac{|b_n - a_n|}{2}$ (for error testing).
4. If $f(a_n) f(m_n) < 0$, then set $a_{n+1} = a_n$ and $b_{n+1} = m_n$.
5. If $f(a_n) f(m_n) > 0$, then set $a_{n+1} = m_n$ and $b_{n+1} = b_n$.

EX 1: Approximate the real root to 2 decimal places. $f(x) = x^4 + 5x^3 + 1$ on $[-1,0]$
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② Newton’s Method Algorithm
Let \( f(x) \) be a differentiable function and let \( x_1 \) be an initial approximation to the root, \( r \) of \( f(x) = 0 \). Let \( E \) denote a bound for the error \( |r - x_n| \).
Repeat the following step for \( n = 1, 2, \ldots \) until \( |x_{n+1} - x_n| < E \).

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

Pros:

Cons:

EX 2 Use Newton’s method to approximate a root of \( 7x^3 + 2x - 5 = 0 \) to 5 decimal places.
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Warning on Newton’s Method: