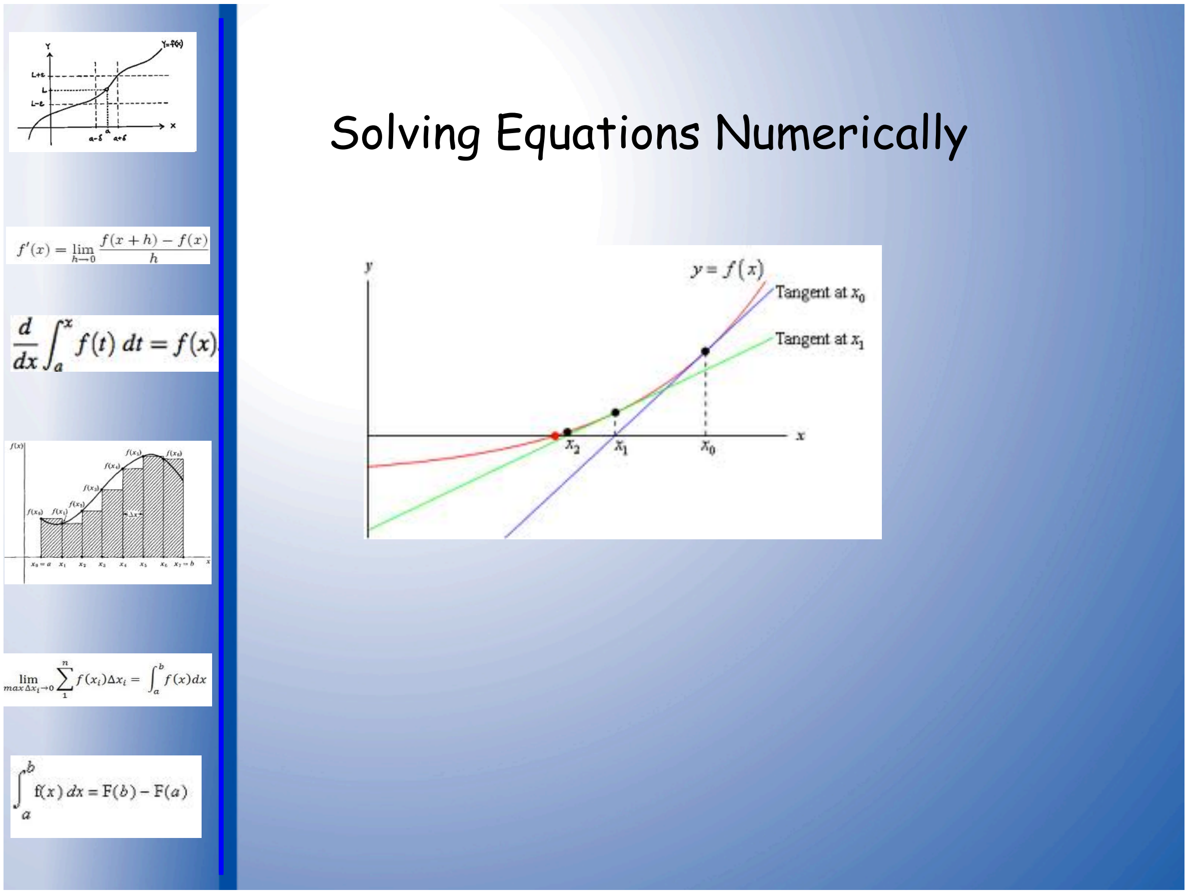
21 Numerical Solutions



Three numeric methods for solving an equation numerically:

① Bisection Method

② Newton's Method

③ Fixed-point Method

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① Bisection Method Algorithm

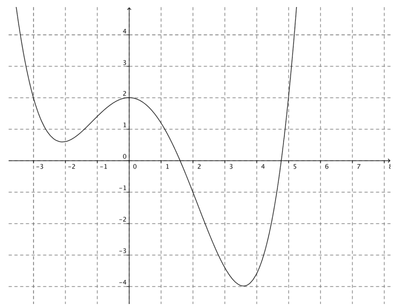
Let *f(x)* be a continuous function and let a_1 and b_1 be numbers satisfying $a_1<b_1$ and $f(a_1) \cdot f(b_1)<0$.

Let *E* denote the desired bound for the error $|r-m_n|$.

Repeat steps *1* to *5* for *n=1,2,...* until $h_n < E$

Pros:

Cons:



1. Calculate $m_n = \dfrac{a_m + b_m}{2}$

2. Calculate $f(m_n)$ and if $f(m_n) = 0$ then STOP.

3. Calculate $h_n = \left|\dfrac{b_n - a_n}{2}\right|$ (for error testing).

4. If $f(a_n) \cdot f(m_n) < 0$, then set $a_{n+1} = a_n$ and $b_{n+1} = m_n$.

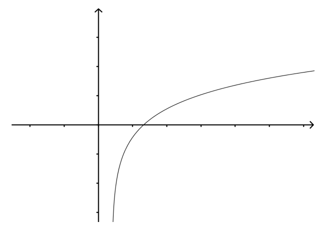
5. If $f(a_n) \cdot f(m_n) > 0$, then set $a_{n+1} = m_n$ and $b_{n+1} = b_n$.

EX 1: Approximate the real root to 2 decimal places. $f(x) = x^4 + 5x^3 + 1 \quad on \quad [-1,0]$

② Newton's Method Algorithm

Let *f(x)* be a differentiable function and let x_1 be an initial approximation to the root, *r* of *f(x) = 0*. Let *E* denote a bound for the error $|r-x_n|$.
Repeat the following step for *n= 1,2,..* until $|x_{n+1}-x_n|<E$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



Pros:

Cons:

EX 2 Use Newton's method to approximate a root of $7x^3+2x-5=0$ to 5 decimal places.

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Warning on Newton's Method: