Calculus: The Slope of a Line

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\frac{d}{dx} \int_a^x f(t) \, dt = f(x)$

$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x_i = \int_a^b f(x) \, dx$

$\int_a^b F(x) \, dx = F(b) - F(a)$
There is only one line between any 2 points.

The slope of a line is:

- The steepness of the line. (same steepness everywhere on the line)
- The vertical change over the horizontal change, denoted by \( m \).

Given two points, \( (x_1,y_1), (x_2,y_2) \) in the Cartesian Plane,

\[
m = \frac{y_2-y_1}{x_2-x_1}
\]

Examples of slope:

EX 1

a) Find the slope of the line containing these points: \((-3,2)\) and \((2,5)\)

\[
m = \frac{y_2-y_1}{x_2-x_1} = \frac{5-2}{2-(-3)} = \frac{3}{5}
\]

b) Find the slope of the line containing these points: \((5,-6)\) and \((-2,-6)\)

\[
m = \frac{-6-(-6)}{-2-5} = \frac{0}{-7} = 0
\]
Point-Slope Form of a Line

Given that \( m \) is the slope of a line and it goes through the point \((x_1, y_1)\), then we know:

\[
y - y_1 = m(x - x_1)
\]

Slope-Intercept Form of a Line

Given that the slope of a line is \( m \) and the y-intercept is the point \((0,b)\), then the equation of the line is:

\[
y = mx + b
\]

EX 2

a) Find the equation of the line going through \((-4, 1)\) and \((5, 2)\).

\[
m = \frac{2 - 1}{5 - (-4)} = \frac{1}{9}
\]

\[
\text{pt. slope: } y - 1 = \frac{1}{9}(x - 4)
\]

\[
y - 1 = \frac{1}{9}x + \frac{4}{9} \Rightarrow y = \frac{1}{9}x + \frac{13}{9}
\]

b) Find the equation of the line with slope, \( m = 3 \) and y-intercept \((0,5)\).

\[
\text{slope-intercept form: } y = 3x + 5
\]
General Equation of a Line

Every line can be written in the form $Ax + By + C = 0$, where $A, B,$ and $C$ are integers.

(I prefer slope-intercept form.)

EX 3

Write the equations from Exercise 2 in general form.

1) $y = \frac{1}{3}x + \frac{13}{9}$
   
   $9y = 9\left(\frac{1}{3}x + \frac{13}{9}\right)$
   
   $9y = x + 13$
   
   $-x + 9y - 13 = 0$
   
   or $x - 9y + 13 = 0$

2) $y = 3x + 5$
   
   $-3x + y - 5 = 0$
   
   or $3x - y + 5 = 0$
Parallel and Perpendicular Lines

Parallel lines have the same slope.

Perpendicular lines have negative reciprocal slopes.

\[
\begin{align*}
\frac{y_1 - y_0}{x_1 - x_0} &= \frac{y_2 - y_0}{x_2 - x_0} \\
\Rightarrow \frac{y_1 - y_0}{x_1 - x_0} &= \left(\frac{y_2 - y_0}{x_2 - x_0}\right)^{-1} \\
\Rightarrow \frac{y_1 - y_0}{x_1 - x_0} &= \frac{1}{m_B} \\
\Rightarrow m_B &= \frac{1}{m_A}
\end{align*}
\]
EX4

a) Find the equation of the line parallel to $3x - 4y = 8$ which passes through the point $(1,3)$.

\[ m = \frac{3}{4} \]

\[ y - 3 = \frac{3}{4}(x-1) \]

\[ y = \frac{3}{4}x - \frac{3}{4} + 3 \]

\[ y = \frac{3}{4}x + \frac{9}{4} \]

\[ \Rightarrow -4y = -3x + 8 \]

\[ y = \frac{3}{4}x - 2 \]

\[ \Rightarrow \text{slope} = \frac{3}{4} \]

b) Find the equation of the line perpendicular to $y = -3x + 5$ which passes through the origin.

\[ m = \frac{1}{3} \]

\[ y = \frac{1}{3}x + 0 \]

\[ \Rightarrow y = \frac{1}{3}x \]

\[ \text{slope} = -3 \]

\[ \perp \text{slope} = -\left(\frac{1}{3}\right) = \frac{1}{3} \]
Determine the slope of each line segment in this function.