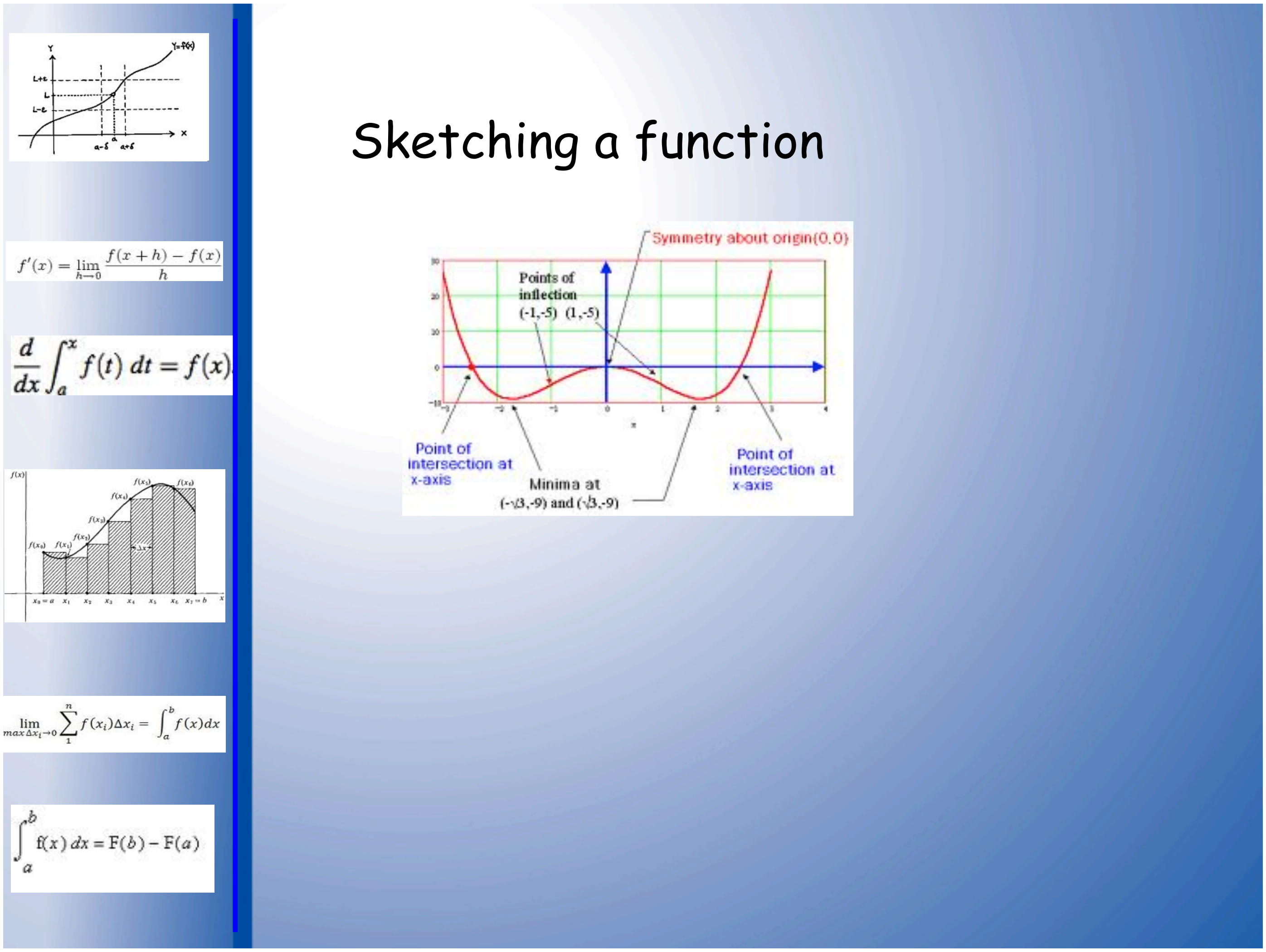
19B Curve Sketching



Sketching a function

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$

$\lim_{\max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$

$\int_a^b f(x)\, dx = F(b) - F(a)$

Symmetry about origin(0,0)

Points of inflection (-1,-5) (1,-5)

Point of intersection at x-axis

Minima at (-√3,-9) and (√3,-9)

Point of intersection at x-axis

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EX 1 Sketch the graph of $f(x)=x^2(x^2-1)$.

a) domain $x \in \mathbb{R}$

(or $(-\infty,\infty)$)

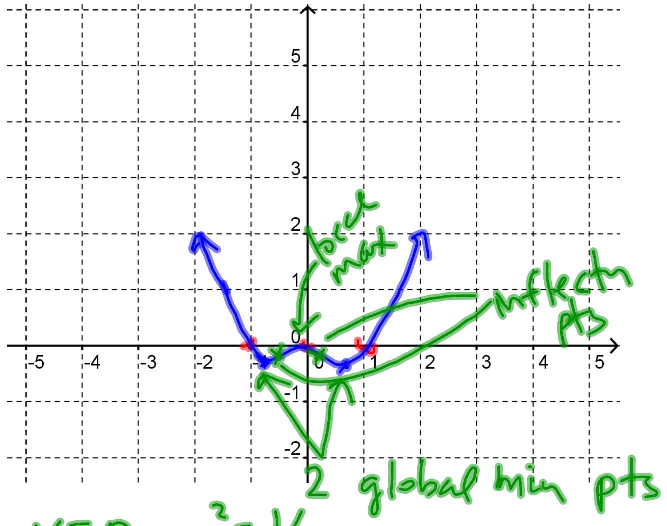
b) symmetry f(x) is even fn $(f(x)=f(-x))$

sym. wrt y-axis

c) x-intercepts

$0 = x^2(x^2-1) \Rightarrow x = 0, \pm 1$

(0,0) (1,0) & (-1,0)



d) First derivative information

$f'(x) = 4x^3 - 2x = 0$

$\boxed{2x(2x^2-1)} = 0 \Rightarrow x=0,\ x^2 = 1/2$

$x = \pm 1/\sqrt{2}$

− + − +

f'(x)

$\frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$ 0 $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

e) Second derivative information

$f''(x) = \boxed{12x^2-2} = 0$

$x^2 = \frac{1}{6}$, $x = \pm\frac{1}{\sqrt{6}}$

f) Asymptotes: none

+ − +

f''(x)

up $\frac{-1}{\sqrt{6}}$ down $\frac{1}{\sqrt{6}}$ up

x values for inflectn pts

$\left(\pm\frac{\sqrt{2}}{2}, \frac{-1}{4}\right)$ min pts

$f\left(\pm\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}\right)^4 - \left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$

(0,0) max pts

$\left(\pm\frac{1}{\sqrt{6}}, \frac{-5}{36}\right)$ inflectn pts

$f\left(\frac{\pm 1}{\sqrt{6}}\right) = \left(\frac{\pm 1}{\sqrt{6}}\right)^4 - \left(\frac{\pm 1}{\sqrt{6}}\right)^2$

$= \frac{1}{36} - \frac{1}{6} = \frac{-5}{36}$

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EX 2 Sketch the graph of $f(x)=\frac{(x-3)^2}{x}$. or $f(x)=\frac{x^2-6x+9}{x}=\frac{x^2}{x}-\frac{6x}{x}+\frac{9}{x}$

or $f(x)=x-6+\frac{9}{x}$

$f(x)=x-6+\frac{9}{x}$

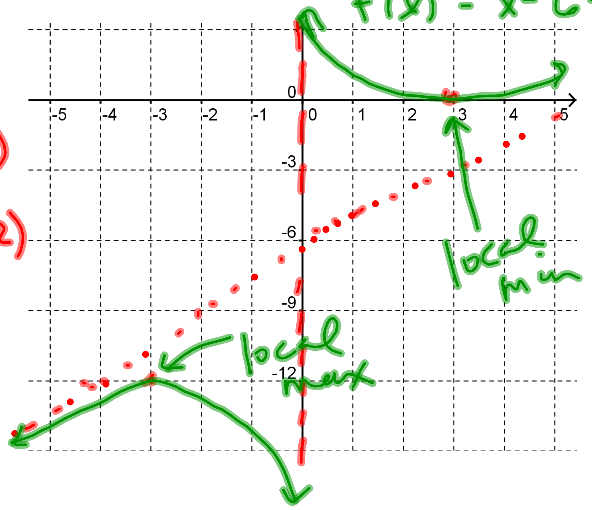
VA: $x=0$

domain: $x\in\mathbb{R}, x\neq 0$

slant asymptote: $y=x-6$

(3,0)

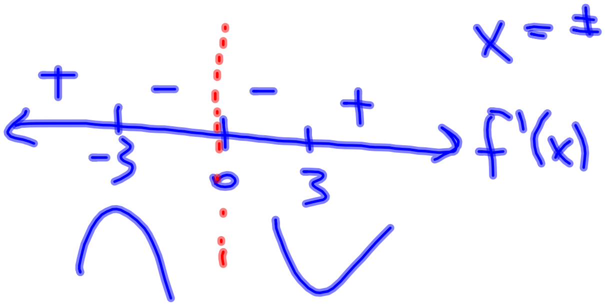
(-3,-12)



$f'(x)=1-\frac{9}{x^2}=0$

$\frac{x^2-9}{x^2}=0 \Leftrightarrow \boxed{\frac{(x-3)(x+3)}{x^2}}=0$

$x=\pm 3$



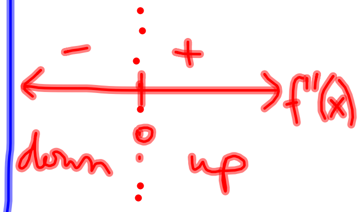
f'(x) test:

$x=-10, \frac{-(-)}{+}$ $x=1, \frac{-(+)}{+}$

$x=-1, \frac{-(+)}{+}$ $x=10, \frac{+(+)}{+}$

$f''(x)=\frac{-9(-2)}{x^3}$

$=\boxed{\frac{18}{x^3}}\neq 0\ (x\neq 0)$



(Is x=0 the x-value for an inflectn pt?)

no because x=0 is VA!!!

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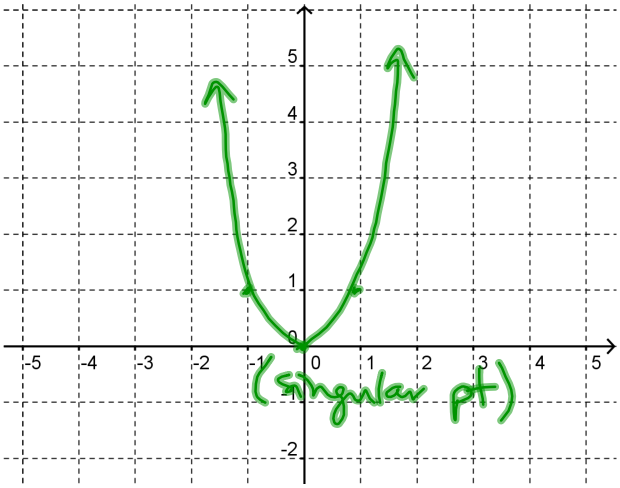
EX 3 Sketch the graph of $f(x)=|x|^3$.

using algebra:

- domain: $x \in \mathbb{R}$
- no VA or HA
- goes thru (0,0)
- only has + (pos) y-values
- symmetric wrt y-axis (even fn)



$f(x)=|x|^3$

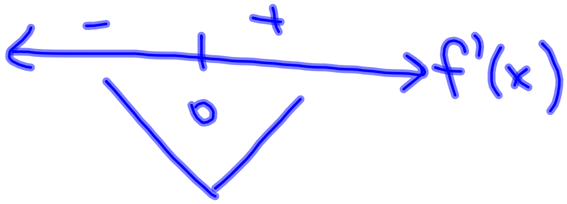
$f'(x)=3|x|^2\left(\frac{x}{|x|}\right)$

$=\boxed{3x|x|}=0 \Rightarrow x=0$

note: $D_x(|x|)$

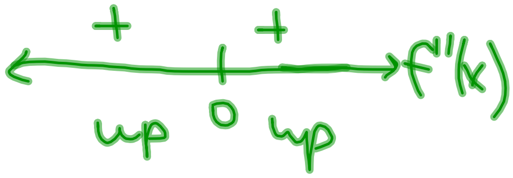
$=\frac{x}{|x|}$

problem at $x=0$ (singular pt)



$f''(x)=3|x|+3x\left(\frac{x}{|x|}\right)$ $(x\neq 0)$

$=\frac{3|x|^2}{|x|}+\frac{3x^2}{|x|}=\frac{6x^2}{|x|}$



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