Definition

Let \( f \) be defined on an interval \( I \), (open, closed or neither), we say that:

1) \( f \) is **increasing** on \( I \) if for every \( x_1, x_2 \in I \), \( x_1 < x_2 \) implies \( f(x_1) < f(x_2) \).
2) \( f \) is **decreasing** on \( I \) if for every \( x_1, x_2 \in I \), \( x_1 < x_2 \) implies \( f(x_1) > f(x_2) \).
3) \( f \) is **strictly monotonic** on \( I \) if it is either increasing or decreasing on \( I \).

**Monotonicity Theorem**

Let \( f \) be continuous on the interval \( I \) and differentiable everywhere inside \( I \).

1) If \( f'(x) > 0 \) for all \( x \) on the interval, then \( f \) is increasing on that interval.
2) If \( f'(x) < 0 \) for all \( x \) on the interval, then \( f \) is decreasing on that interval.
17 Monotonicity Concavity

EX 1  For each function, determine where $f$ is increasing and decreasing.

a) $f(x) = x^3 + 3x^2 - 12$

b) $f(x) = \frac{x-1}{x^2}$

EX 2  Where is $f(x) = \cos^3 x$ increasing and decreasing on the interval $[0, 2\pi]$?
17 Monotonicity Concavity

**Definition**

Let $f$ be differentiable on an open interval, $I$. $f$ is concave up on $I$ if $f'(x)$ is increasing on $I$, and $f$ is concave down on $I$ if $f'(x)$ is decreasing on $I$.

![Graphs of concave up and concave down functions]

**Concavity Theorem**

Let $f$ be twice differentiable on an open interval, $I$. If $f''(x) > 0$ for all $x$ on the interval, then $f$ is concave up on the interval. If $f''(x) < 0$ for all $x$ on the interval, then $f$ is concave down on the interval.

**EX 3** Determine where this function is increasing, decreasing, concave up and concave down.

$$f(x) = 4x^4 - 3x^2 - 6x + 12$$
Inflection Point

Let \( f \) be continuous at \( c \). We call \((c, f(c))\) an inflection point of \( f \) if \( f \) is concave up on one side of \( c \) and concave down on the other side of \( c \).

Inflection points will occur at \( x \)-values for which \( f''(x) = 0 \) or \( f''(x) \) is undefined.

EX 4 For this function, determine where it is increasing and decreasing, where it is concave up and down, find all max/min and inflection points.

Use this information to sketch the graph.

\[
f(x) = 8x^{3/3} - x^{4/3}
\]