Math 1090 ~ Business Algebra

Section 5.5 Loans and Amortization

Objectives:
• Create an amortization schedule for a loan.
• Determine the amount of a loan you can afford given certain conditions.
• Determine the amount of each monthly payment for a given number of years.
**Amortization**: An "installment loan" is a loan that is repaid by making all payments equal.

The bank is basically investing a lump sum of dollars and getting a periodic return which is exactly like PV of an ordinary annuity.

\[
R = S \left( \frac{r_c}{1 - (1+r_c)^{-N}} \right)
\]

Amortization Formula

\[ S = \text{loan amount} \quad R = \text{payment amount} \]

Ex 1: When you graduate college, you buy a new car and can afford a monthly payment of $250/month. If you get a special rate of 3.6% interest, compounded monthly, for 6 years, how much can you afford to borrow?

\[ R = 250, \quad r = 0.036, \quad t = 6 \text{ yrs}, \quad S = ? \]

\[ r_c = \frac{0.036}{12} = 0.003, \quad N = 6(12) = 72 \]

\[
250 = S \left( \frac{0.003}{1 - 1.003^{-72}} \right)
\]

\[
S = 250 \left( \frac{1 - 1.003^{-72}}{0.003} \right)
\]

\[ S \approx \boxed{16,167.01} \]

**Total payments:**

\[ 250(72) = \boxed{\$18,000} \]
Ex 2: Alex buys a house for $200,000. They put $15,000 down and get a loan for the rest at 5.4% interest compounded monthly for 20 years. What will their payments be?

\[ S = \$185,000, \ r = 0.054, \ n = 12, \ t = 20 \text{ yrs} \]

\[ r_c = \frac{0.054}{12} = 0.0045, \ N = 12(20) = 240 \]

\[ R = \frac{185000(0.0045)}{1-1.0045^{-240}} \]

\[ R \approx \$1262.17 \]

**Total payments:**

\[ 1262.17(240) = \$302,920.80 \]

**Amortization Schedule**

A loan of $10,000 with interest rate of 10% could be repaid in 5 equal annual payments.

\[ R = S \left( \frac{r_c}{1-(1+r_c)^{-N}} \right) \]

\[ r_c = 0.1, \ N = 1, \ t = 5 \]

\[ R = 10000 \left( \frac{0.1}{1-0.1} \right) = \$2637.97 \]

<table>
<thead>
<tr>
<th>payment</th>
<th>interest</th>
<th>int + principal = Payment</th>
<th>principal</th>
<th>unpaid balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2637.97</td>
<td>10000(0.1) = 1000</td>
<td>1637.97</td>
<td>8362.03</td>
</tr>
<tr>
<td>2</td>
<td>2637.97</td>
<td>8362.03(0.1) = 836.20</td>
<td>1801.77</td>
<td>6560.26</td>
</tr>
<tr>
<td>3</td>
<td>2637.97</td>
<td>6560.26(0.1) = 656.03</td>
<td>1981.94</td>
<td>4578.32</td>
</tr>
<tr>
<td>4</td>
<td>2637.97</td>
<td>4578.32(0.1) = 457.83</td>
<td>2180.14</td>
<td>2398.18</td>
</tr>
<tr>
<td>5</td>
<td>2637.97</td>
<td>2398.18(0.1) = 239.82</td>
<td>2398.15</td>
<td>0.03</td>
</tr>
</tbody>
</table>

\[ \Rightarrow \text{ your very last payment will actually be } \$2638.00 \ (\text{to cover that leftover } 34) \]

**Total payments:**

\[ 2637.97(5) = \$13,189.85 \]
Ex 3: A company that buys a piece of equipment by borrowing $250,000 for 10 years at 6% compounded monthly has monthly payments of $2,775.51.

a) Find the unpaid balance after 1 year.

\[ t = 10, \quad r = 0.06, \quad n = 12 \]

\[ k = 12, \quad N - k = 108 \]

\[ R = 42,775.51 \]

\[ S_{108} = 2775.51 \left( \frac{1 - 1.005^{-108}}{0.005} \right) \]

\[ S_{108} \approx 231,181.73 \]

b) During that first year, how much interest does the company pay?

\[ 2775.51 \times 12 = 33,306.12 \quad \text{(total payments in 1st year)} \]

\[ \Rightarrow 250,000 - 231,181.73 = 18,818.27 \quad \text{(amt that went toward principal)} \]

\[ \Rightarrow 33,306.12 - 18,818.27 = \frac{14,487.85}{\boxed{14,487.85}} \]

(this is the amount of 1st year of payments that went toward interest)