Math 1090 ~ Business Algebra
Section 5.4 Present Value of Annuities

Objectives:
• Determine the present value of an ordinary annuity.
• Solve problems involving annuities.
• Distinguish between present value and future value word problems.

Present Value of an annuity: We calculate this when we leave a lump sum of dollars in an account and make regular withdrawals (like what happens after a person retires.)

<table>
<thead>
<tr>
<th>ordinary annuity</th>
<th>annuity due</th>
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<tbody>
<tr>
<td>withdrawals occur at the end of each period.</td>
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Ex 1: You want to withdraw $1000 at the end of each year from an account that earns 10% interest compounded annually for 4 years. How much needs to be in the account from the start?

\[ S = P \left(1 + \frac{r}{n}\right)^{nt} \quad r = \frac{r}{n} \]

After 1st year:

After 2nd year:

After 3rd year:

After 4th year:
Present Value of an Ordinary Annuity

\[ P = R \left[ \frac{1 - (1 + r)^{-n}}{r} \right] \]

Present Value of an Annuity Due

\[ P_{\text{due}} = \frac{R(1 + r)(1 - (1 + r)^{-n})}{r} \]

Ex 2: Find PV of an annuity that pays $4000 at the end of each month from an account that earns 8% interest compounded monthly for 25 years.

Ex 3: An inheritance of $500,000 will provide how much at the end of each year for 20 years if money is worth 7.2% compounded annually?

Deferred Annuity: The first payment is deferred until a later date at which point regular payments are made.

\[ P = P_{\text{F}} \text{ of deferred annuity} \quad m = \text{number of periods of deferment} \]

\[ N = \text{number of regular withdrawals} \quad R = \text{payment each period} \]

\[ P = \frac{R(1 - (1 + r)^{-n})}{r(1 + r)^m} \]

Ex 4: Carol received a trust fund inheritance of $10,000 on her 30th birthday. She plans to use it to supplement her income with 20 quarterly payments beginning on her 60th birthday. If money is worth 8.1% compounded quarterly, how much will each payment be?
Ex 5: A lottery prize worth $1,800,000 is awarded in payments of $10,000 at the beginning of each month for 15 years. Suppose money is worth 6.6% monthly. What is the real value of the prize?