Math 1090 ~ Business Algebra

Section 5.4 Present Value of Annuities

Objectives:
• Determine the present value of an ordinary annuity.
• Solve problems involving annuities.
• Distinguish between present value and future value word problems.
Present Value of an annuity: We calculate this when we leave a lump sum of dollars in an account and make regular withdrawals (like what happens after a person retires.)

ordinary annuity

withdrawals occur at the end of each period.

annuity due

withdrawals occur at the beginning of each period.

Ex 1: You want to withdraw $1000 at the end of each year from an account that earns 10% interest compounded annually for 4 years. How much needs to be in the account from the start?

\[ S = P \left(1 + \frac{r}{n}\right)^n \]
\[ r_c = \frac{L}{n} \]
\[ N = nt \]

\[ S = P (1 + r_c)^N \]

\[ S = \frac{1000}{(1 + 0.1)^1} = \$909.09 \]

After 1st year:

\[ P_1 = 1000(1.1)^{-1} = \$909.09 \]

After 2nd year:

\[ P_2 = 1000(1.1)^{-2} = \$826.45 \]

After 3rd year:

\[ P_3 = 1000(1.1)^{-3} = \$751.31 \]

After 4th year:

\[ P_4 = 1000(1.1)^{-4} = \$683.01 \]

\[ P_1 + P_2 + P_3 + P_4 = 1000(1.1)^{-1} + 1000(1.1)^{-2} + 1000(1.1)^{-3} + 1000(1.1)^{-4} \]

\[ \text{this is sum of geom. sequence again!!!} \]
Present Value of an Ordinary Annuity

\[ P = R \left[ \frac{1 - (1 + r_c)^{-N}}{r_c} \right] \]

Present Value of an Annuity Due

\[ P_{\text{due}} = \left[ \frac{R(1 + r_c)(1 - (1 + r_c)^{-N})}{r_c} \right] \]

Ex 2: Find PV of an annuity that pays $4000 at the end of each month from an account that earns 8% interest compounded monthly for 25 years.

\[ PV = \text{lump sum amt. in acct. to produce all these payments} \]
\[ R = 4000, \ t = 25, \ r = 0.08, \ n = 12, \ r_c = \frac{0.08}{12} = 0.006 \]
\[ P = 4000 \left( \frac{1 - 0.006^{-300}}{0.006} \right) \approx 518,258.09, \ N = 12 \times 25 = 300 \]

Total withdrawals from acct:
\[ 4000 \times 25 \times 12 = 1200000 \]

Ex 3: An inheritance of $500,000 will provide how much at the end of each year for 20 years if money is worth 7.2% compounded annually?

\[ n = 1, \ t = 20, \ r = 0.072 \]
\[ r_c = 0.072, \ N = 20 \]
\[ PV \text{ ordinary annuity:} \]
\[ P = R \left[ \frac{1 - (1 + r_c)^{-N}}{r_c} \right] \]
\[ 500000 = R \left[ \frac{1 - 1.072^{-20}}{0.072} \right] \]
\[ R = \frac{500000 \times 0.072}{1 - 1.072^{-20}} \approx 47932.61 \]

Total withdrawals:
\[ 47932.61 \times 20 = 958652.20 \]
Deferred Annuity: The first payment is deferred until a later date at which point regular payments are made.

\[ P = PV \text{ of deferred annuity} \quad m = \text{number of periods of deferment} \]

\[ N = \text{number of regular withdrawals} \quad R = \text{payment each period} \]

\[
P = \frac{R(1-(1+r_c)^{-N})}{r_c(1+r_c)^m}
\]

Ex 4: Carol received a trust fund inheritance of $10,000 on her 30\textsuperscript{th} birthday. She plans to use it to supplement her income with 20 quarterly payments beginning on her 60\textsuperscript{th} birthday. If money is worth 8.1\% compounded quarterly, how much will each payment be?

\[ r = 0.081, \quad n = 4, \quad P = 10,000 \]
\[ r_c = \frac{0.081}{4} = 0.02025, \quad N = 20, \quad m = 30(4) = 120 \]

\[
10,000 = R\left(\frac{1-1.02025^{-20}}{0.02025(1.02025^{20})}\right)
\]

\[
R = \frac{10,000(0.02025(1.02025^{120}))}{(1-1.02025^{-20})}
\]

\[
R \approx \$6,796.47
\]

Total withdrawals:

\[
6796.47(20) = \$135,929.40
\]
Ex 5: A lottery prize worth $1,800,000 is awarded in payments of $10,000 at the beginning of each month for 15 years. Suppose money is worth 6.6% monthly. What is the real value of the prize?

\[
\text{total payments: } \ 10,000(15)(12) = 1,800,000 \ \checkmark
\]

\[
P_{\text{due}} = \frac{P \left(1+r_c\right) \left(1-\left(1+r_c\right)^{-n}\right)}{r_c}
\]

\[
= \frac{10,000 \left(1.0055\right) \left(1-1.0055^{-180}\right)}{0.0055}
\]

\[
\approx \ 1,147,026.90
\]