Math 1090 ~ Business Algebra
Section 5.2 Simple and Compound Interest

Objectives:
• Differentiate between simple and compound interest.
• Solve problems involving simple and compound interest.
• Understand and calculate annual percentage yield (APY).

\[ APY = \left(1 + \frac{r}{n}\right)^n - 1 \]
Simple and Compound Interest

**Simple Interest**
- add same interest every period
- arithmetic sequence (grows linearly)
- balance is the sum
- $P = \text{principal} = \text{start value}$
- $Pr = \text{principal times interest rate}$

$$S = P + Pr(t)$$

**Simple Interest formula**

$P = \text{principal}$

$r = \text{annual interest rate}$

$t = \text{number of years}$

$S = \text{future account value}$

(\text{acct value after } t\ \text{yrs})

**Compound Interest**
- multiply by same rate every period
- geometric sequence (grows exponentially)
- balance is the sum
- $P = \text{principal} = \text{start value}$
- $(1 + r) = \text{factor that's multiplied by principal every year}$

\[ S = P(1 + r)^t \]

**Compound interest formula**

If we compound $n$ times per year,

\[ S = P \left(1 + \frac{r}{n}\right)^{nt} \]

**Continuous compounding**

\[ S = Pe^{rt} \]
Ex 1: If $10,000 is invested for four years at an annual rate of 8%, how much will the account be worth at the end of four years?

a) simple interest \[ P = 10,000 \]
\[ t = 4 \]
\[ r = 0.08 \]
\[ S = P(1+rt) \]
\[ S = 10000(1+0.08(41)) \]
\[ S = 13,200 \]

b) compounded once a year
\[ S = P(1+\frac{r}{n})^{nt} \]
\[ n = 1 \]
\[ S = 10000(1+\frac{0.08}{1})^{1\cdot4} \]
\[ S = 10000(1+0.08)^4 \]
\[ S = 13,604.89 \]
Ex 2: What is an account worth in 8 years if we started with $3000 and we got continuous compounding at a rate of 6%?

\[ S = Pe^{rt} \]

\[ S = 3000e^{0.06(8)} \approx 4848.22 \]

Ex 3: If $1000 is invested at 5% annual interest rate, compute these.

<table>
<thead>
<tr>
<th>Simple Interest</th>
<th>Compound Interest, n = 1</th>
<th>Compound Interest, n = 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ S = P(1 + rt) ]</td>
<td>[ S = 1000(1 + 0.05(5)) ]</td>
<td>[ S = 1000(1 + \frac{0.05}{12})(12t) ]</td>
</tr>
<tr>
<td>[ S = 1250 ]</td>
<td>[ S \approx 1,276.28 ]</td>
<td>[ S \approx 1,283.36 ]</td>
</tr>
</tbody>
</table>

Balance after 5 years: \( t = 5 \)

How long to double investment: \( S = 2000 \)

\[ 2000 = 1000(1 + 0.05t) \]

\[ 2 = 1 + 0.05t \]

\[ t = \frac{2}{0.05} = 40 \text{ yrs} \]

\[ 2000 = 1000 \left( 1 + \frac{0.05}{12} \right)^{12t} \]

\[ 2 = \left( 1 + \frac{0.05}{12} \right)^{12t} \]

\[ \ln 2 = \ln \left( 1 + \frac{0.05}{12} \right)^{12t} \]

\[ t = \frac{\ln 2}{\ln \left( 1 + \frac{0.05}{12} \right)} \]

\[ t = \frac{\ln 2}{0.05} \approx 13.86 \text{ yrs} \]
Ex 4: What amount must be invested now in order to have $1,000,000 for retirement in 45 years if money is compounded quarterly at 9%?

\[ S = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ 1000000 = P \left( 1 + \frac{0.09}{4} \right)^{4(45)} \]

\[ 1000000 = P \left( 1.0225 \right)^{180} \]

\[ P = \frac{1000000}{1.0225^{180}} \approx 18,222.29 \]

**APY** (Annual Percentage Yield)

Let \( P = 100 \) be invested at 8% interest compounded as given in (a) and (b). What is the account worth after one year?

a) quarterly \( n = 4 \) \( t = 1 \)

\[ S = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ S = 100 \left( 1 + \frac{0.08}{4} \right)^{4(1)} \]

\[ S \approx 108.24 \]

\[ \Rightarrow \text{APY} = 8.24\% \]

\[ \text{APR} = r = 8\% \]

b) monthly \( n = 12 \)

\[ S = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ S = 100 \left( 1 + \frac{0.08}{12} \right)^{12(1)} \]

\[ S \approx 108.30 \]

\[ \text{APY} = 8.3\% \]

\[ \text{APR} = r = 8\% \]
\[ APY = \left(1 + \frac{r}{n}\right)^n - 1 \quad \text{(periodic compounding)} \]
\[ APY = e^r - 1 \quad \text{(continuous compounding)} \]

Ex 5: Which is a better investment deal?

a) 10% compounded annually \( n=1, \ r=0.1 \)
\[
APY = \left(1 + \frac{0.1}{1}\right)^1 - 1 = 0.1 = 10\% 
\]

b) 9.8% compounded quarterly \( n=4, \ r=0.098 \)
\[
APY = \left(1 + \frac{0.098}{4}\right)^4 - 1 \approx 0.10166 \approx 10.166\% \text{ deal} 
\]

c) 9.65% compounded continuously \( r=0.0965 \)
\[
APY = e^{0.0965} - 1 \approx 0.10131 = 10.131\% 
\]