Math 1090 ~ Business Algebra

Section 3.2 Parabolas: Quadratic Equations in Two Variables

Objectives:
• Identify a quadratic function, including the dependent and independent variables.
• Sketch a graph of a quadratic function.
• Identify the vertex, the axis of symmetry, concavity, y-intercept and roots of a quadratic function.

\[ 5x - 2y \leq 75 \]

\[ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \]

\[ S = Pe^t \]

\[ APY = \left(1 + \frac{r}{n}\right)^n - 1 \]
A quadratic function in two variables can be written in the form

\[ y = f(x) = ax^2 + bx + c \quad a \neq 0, \quad a, b, c \in \mathbb{R} \]

The independent variable \( x \) is

\[ a, b, c \text{ constants} \]

How can we find the vertex?

Let \( y = c \). We get

\[ c = ax^2 + bx + c \]
\[ 0 = ax^2 + bx \]
\[ 0 = x(ax + b) \]

\[ x = 0 \quad \text{or} \quad ax + b = 0 \]

\[ x = \frac{-b}{a} \]

\( \Rightarrow \) vertex is halfway between \( x = 0 \) and \( x = \frac{-b}{a} \)

i.e. vertex occurs when

\[ x = \frac{1}{2} \left( \frac{-b}{a} \right) = \frac{-b}{2a} \]

vertex: \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \)

Check: plug in \( x = 0 \)

\[ \Rightarrow y = a(0) + b(0) + c = c \]

So parabola goes through pt \((0, c)\).
Ex 1: For \( y = -2x^2 - 4x + 6 \) 

a) Find the vertex.

\[
a = -2, \quad b = -4
\]

\[
x = \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = -1
\]

\[
y = -2(-1)^2 - 4(-1) + 6 = -2 + 4 + 6 = 8
\]

Vertex: \((-1, 8)\)

b) Is the vertex a min or max point?

\[
a = -2 < 0 \implies \text{parabola is concave down}
\]

\[\Rightarrow \text{vertex is max pt}\]
Ex 2: For \( y = x^2 - 6x + 9 \),

\[ a = 1, \ b = -6, \ c = 9 \]

a) Find the vertex.
\[ x = \frac{-b}{2a} = \frac{6}{2} = 3 \]
\[ y = 3^2 - 6(3) + 9 = 0 \]

(3, 0)

b) Is it a min or max point?
\[ a = 1 > 0 \] vertex is min

\[ a = 1 > 0 \] vertex is min

\( \cup \)

Is it a min or max point?

\[ y = x^2 - 6x + 9 \]
\[ 0 = (x - 3)(x - 3) \]
\[ x - 3 = 0 \]  \( \iff \) \[ x = 3 \]

d) Find the axis of symmetry

\[ x = 3 \]

e) Find the y-intercept.
\[ y = 0^2 - 6(0) + 9 = 9 \]
\[ (0, 9) \]

f) Sketch the graph
Ex 3: For $y = -x^2 + 4x + 5$, 

\[a = -1, \ b = 4, \ c = 5\]

a) Find the vertex.

\[x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2\]

\[y = -(2^2) + 4(2) + 5 = -4 + 8 + 5 = 9\]

\[\text{(2,9)}\]

\[\text{vertex}\]

b) Is this parabola concave up or concave down?

\[q = -1 < 0 \Rightarrow \text{concave down}\]

c) Find the x and y-intercepts of the graph.

\[\begin{align*}
\text{x-int:} & \quad (-1, 0), (5, 0) \\
0 &= -x^2 + 4x + 5 \\
0 &= (x - 5)(x + 1) \\
x &= 5, -1
\end{align*}\]

\[\begin{align*}
\text{y-int:} & \quad (0, 5) \\
y &= 0 + 5 = 5
\end{align*}\]

d) Find the axis of symmetry 

\[x = 2\]

e) Sketch the graph
Ex 4: For the parabola from example 1, \( y = -2x^2 - 4x + 6 \), sketch the graph.

vertex \((-1, 8)\)
concave down

Coefficient of \(x^2\) is -2
Ex 5: If 100 ft of fencing is used to enclose a rectangular yard, find the area function. Find the dimensions of the rectangle that maximizes the area.

\[ P = 100 \text{ ft} = 2x + 2y \]

\[ 100 = 2x + 2y \]

\[ 50 = x + y \]

\[ y = 50 - x \]

\[ A = A(x) = xy \]

\[ A = x(50-x) \]

\[ A(x) = 50x - x^2 \]

\[ A(x) = -x^2 + 50x \]

⇒ area is a quadratic fn of x.

and leading coefficient is negative

⇒ we have concave down parabola

⇒ max area occurs at vertex

\[ \text{vertex: } x = \frac{-b}{2a} = \frac{-50}{2(-1)} = 25 \]

\[ \text{dimensions of rectangle: } x = 25, y = 50 - 25 = 25 \]

\[ 25 \text{ ft} \times 25 \text{ ft} \]